PHILOSOPHY 512:
MODAL LOGIC

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MWF 11-11:50, SPRING 2024

PROBLEM SETS

PROBLEM SET #1
Due: Monday 29 January at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) logical connectives, (b) translations, (c) replacement rules, (d) inference rules, and (e) the Konyndyk reading.

Problem 2 (3 points). For each of the following three arguments, please (a) translate the argument into logical notation, using the provided symbols, and then (b) if the translated argument is valid, prove its validity by natural deduction, but if it is invalid, prove its invalidity by the tableau method. Each argument is worth 1 point (0.5 for the translation and 0.5 for the proof).

1. The school pays if and only if is not the case that both the school makes an offer and the applicant refuses the job. Therefore, the job's being both offered and refused is a necessary condition for the school not to pay. (*P*, *O*, *R*)
2. Murders are committed and regretted. If murder is not bad, then judgments of regret are mistaken. If determinism is true, then if murders are committed and are bad, sin is a necessary part of the world. If determinism is true, then if murders are regretted and judgments of regret are mistaken, error is a necessary feature of the world. Hence, if determinism is true, then either sin or error is a necessary part of world. (*C*, *R*, *B*, *M*, *D*, *S*, *E*)
3. If the union wins the election and establishes an agency shop, then you will have to pay a fee of $135 per year or you will lose your job. If the union wins the election and does not establish an agency shop but requires membership in order to vote, then you will have to pay a fee of $135 per year or you will be disenfranchised. If the union wins the election and neither establishes an agency shop nor requires membership in order to vote, then its power base will be undercut, and the union will not be effective. Therefore: if the union wins the election and is effective, then you will lose your job or be disenfranchised unless you pay a fee of $135 per year. (*W*, *E*, *P*, *R*, *D*, *U*, *F*)

PROBLEM SET # 2
Due: Monday 5 February at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) the material of the previous quiz, (b) semantics and semantic tableaux for K, and (c) the Loux reading.

Problem 2 (1 point). Consider the frame $<\{w_0, w_1, w_2, w_3\}, \{Rw_0w_3, Rw_1w_2, Rw_3w_1, Rw_2w_2, Rw_3w_2\}>$, and suppose *A* is true at w_0 and w_1 and false at w_2 and w_3, *B* is true at w_0 and w_2 and false at w_1 and w_3, and *C* is true at w_0 and w_3 and false at w_1 and w_2. Find the truth values, at every world, of the following two sentences:

1. $\Diamond\Box({\sim}A \;\&\; \Diamond C)$
2. $\Box\Diamond\Box\,(A \;\&\; \Box\Diamond\Diamond(B \rightarrow {\sim}\Diamond C))$

Problem 3 (2 points). For each of the following two arguments, please (a) translate the argument into logical notation, using the provided symbols, and then (b) prove the validity or the invalidity (in K) of the translated argument by the tableau method. Each argument is worth 1 point (0.5 for the translation and 0.5 for the proof).

1. It's possibly necessary that if God doesn't exist in reality then there's a being greater than God. It's necessarily impossible that there be a being greater than God [since "God" is defined as "a being than which no being could be greater"]. Therefore, it's possibly necessary that God exists in reality. (*R*: God exists in reality; *B*: there's a being greater than God.)
2. Necessarily, this is the best of all possible worlds. For, necessarily, God exists; and, necessarily, if God is not both morally perfect and omnipotent, then God does not exist. Necessarily, if God is omnipotent, it is possible for God to create just any possible world. And, necessarily, if God is morally perfect, God will create the best possible world if it is possible for her to create it. Moreover, necessarily, it is possible for God to create the best of all possible worlds if and only if it is possible for God to create just any possible world. Finally, necessarily, this is the best of all possible worlds given that God will create the best of all possible worlds. (*G*: God exists; *M*: God is morally perfect; *O*: God is omnipotent; *A*: God creates just any possible world; *W*: God will create the best of all possible worlds; *B*: This the best of all possible worlds.)

PROBLEM SET #3
Due: Monday 12 February at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) natural deduction for K and (b) the (new) Loux reading.

Problem 2 (3 points). Please translate the following three arguments into logical notation, using the provided symbols, and then prove their K-validity by natural deduction. Each argument is worth 1 point (0.5 for the translation and 0.5 for the proof).

1. If it is possible that God is either omnipotent or perfectly good, that it is necessary that God both exists and is omniscient. Possibly, God is both omnipotent and omniscient. Therefore, God possibly exists. (*P*: God is omnipotent; *G*: God is perfectly good; *E*: God exists; *S*: God is omniscient.)
2. Necessarily, God's existence is a matter of metaphysical luck if it is contingent. But it is impossible for God's existence to be a matter of metaphysical luck. Necessarily, God's existence is not impossible if it is possible for an omnipotent and perfectly good being to exist. It is possibly possible for an omnipotent and perfectly good being to exist. Therefore, God's existence is possibly necessary. (*G*: God exists; *L*: God's existence is a matter of metaphysical luck; *O*: an omnipotent and perfectly good being exists.)
3. Possibly, it is impossible for God to know the future free acts of her creatures if God is in time. For, necessarily, if God is in time, God's knowledge of the future is a prediction based on the past and present. However, necessarily, if humans have free will, then it is impossible to infallibly predict their future acts based on the past and the present. And, necessarily, if it is impossible to infallibly predict the future acts of humans based on the past and the present, then it is impossible for God to know the future free acts of her creatures if God is in time. Finally, it is possible that, if humans do not have free will, then God's knowledge of the future is not a prediction based on the past and the present. (*T*: God is in time; *P*: God's knowledge of the future is a prediction based on the past and present; *F*: Humans have free will; *I*: The future acts of humans are infallibly predicted based on the past and present; *K*: God knows the future free acts of her creatures.)

PROBLEM SET #4
Due: Monday 19 February at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) semantics and semantic tableaux for extensions of K and (b) the Salmon reading.

Problem 2 (3 points). For each of the following two arguments, please (a) translate the argument into logical notation, using the provided symbols, and then (b) prove the validity of the translated argument in *a weakest normal system in which it is valid but without assuming transitivity* by the tableau method. Each argument is worth 1.5 point (0.5 for the translation and 1.0 for the proof).

1. Necessarily, God is both possibly perfectly good and necessarily both omnipotent and omniscient. Therefore: necessarily, either God is possibly not perfectly good or it is necessarily necessary that God is either omniscient or nonexistent. (*P*: God is omnipotent; *G*: God is perfectly good; *E*: God exists; *S*: God is omniscient.)
2. Necessarily, if it is possible that God exists then it is necessary that God necessarily exists. Possibly, God does not exist. Therefore, God does not exist. (*E*: God exists.)

PROBLEM SET #5
Due: Monday 26 February at 11:00am in class

Problem 1 (1 point). In-class quiz on natural deduction for extensions of K.

Problem 2 (3 points). Please prove by natural deduction (a) the validity of the first translated argument (*in a weakest normal system in which it is valid but without assuming transitivity*) in Problem 2 of Problem Set #4 and (b) the validity of the argument from A to $\Box A$ in K$\rho\sigma^*$, where σ^* is the inference rule specifying that from p one can derive $\Diamond\Box p$. Each argument is worth 1.5 point (0.0 for the translation and 1.5 for the proof).

PROBLEM SET #6
Due: Wednesday 6 March at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) classical quantified logic and (b) the Loux reading.

Problem 2 (3 points). Please (a) translate the following two arguments into logical notation, using the provided symbols, and then (b) prove the validity of the translated arguments by using the tableau method for the first argument and natural deduction for the second argument. Each argument is worth 1.5 point (0.5 for the translation and 1.0 for the proof).

1. A theory that does not render the concept of justice meaningful is not really a theory of that concept. A theory renders the concept of justice meaningful only if for every criminal act the theory assigns legal consequences that are fairly proportionate to the act itself. However, there are criminal acts for which the therapeutic theory of justice does not assign legal consequences that are fairly proportionate to the act itself. Therefore, the therapeutic theory of justice, if it is a theory at all, is not really a theory of the concept of justice. (*Tx*: *x* is a theory; *Rx*: *x* renders the concept of justice meaningful; *Jx*: *x* is really a theory of the concept of justice; *Cx*: *x* is a criminal act; *Axy*: *x* assigns to *y* legal consequences that are fairly proportionate to *y* itself; *t*: the therapeutic theory of justice.)
2. Any two men either love each other or hate each other. Therefore, either there is a man who loves all men or for every man there exists some man whom he hates. (*Mx*: *x* is a man; *Lxy*: *x* loves *y*, *Hxy*: *x* hates *y*.)

PROBLEM SET #7
Due: Wednesday 13 March at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) free logic, (b) semantics for quantified modal logic, (c) the Barcan formulas, and (d) the required readings for Part II.B of the course (Forbes, Plantinga, Bennett).

Problem 2 (3 points). For each of the following two arguments, please (a) translate the argument into logical notation, using *restricted* quantifiers and the provided symbols, and then, *in (positive) free logic*, (b) if the translated argument is valid, prove its validity by natural deduction, but if it is invalid, prove its invalidity by the tableau method. Each argument is worth 1.5 point (0.5 for the translation and 1.0 for the proof).

1. No one respects anyone who does not respect herself. No one will hire anyone she does not respect. Therefore, anyone who respects no one will never be hired by anyone. (*Rxy*: *x* respects *y*; *Hxy*: *x* hires *y*.)
2. A work of art that tells a story can be understood by everyone. Some religious works of art have been created by great artists. Every religious work of art tells a story. Someone admires only what she cannot understand. Therefore, some great artists' creations will not be admired by everyone. (*Gx*: *x* is a great artist; *Sx*: *x* is a story; *Rx: x* is religious; *Wx*: *x* is a work of art; *Cxy*: *x* creates *y*; *Axy*: *x* admires *y*; *Txy*: tells *y*; *Uxy*: *x* can understand *y*.)

PROBLEM SET #8
Due: Wednesday 20 March at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) semantic tableaux and natural deduction for non-restricted quantifiers and (b) the Fitting et al. and Salmon readings.

Problem 2 (3points). For each of the following two arguments, please (a) translate the argument into logical notation, using *non-restricted* quantifiers and the provided symbols, and then (b) if the translated argument is invalid in QKυ, prove its invalidity by the tableau method, but if it is valid, prove by natural deduction its validity *in a weakest normal extension of QK in which it is valid*. Each argument is worth 1.5 point (0.5 for the translation and 1.0 for the proof).

1. Possibly, I exist but nothing material exists. Whatever is material is *essentially* material (i.e., necessarily, it is material if it exists). Therefore, I am not material. (*Mx*: *x* is material; *i*: I.)
2. Some existing property has a contingently existing individual as a constituent. If a property has an individual as a constituent, then, necessarily, if the property exists then it has that individual as a constituent. Necessarily, a property exists only if all its constituents exist. Every property is *essentially* a property. Therefore, some property exists contingently. (*Px*: *x* is a property; *Cxy*: *x* is a constituent of *y*.)

PROBLEM SET #9
Due: Monday 1 April at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) semantic tableaux and natural deduction for restricted quantifiers and (b) the Loux and Sider readings.

Problem 2 (3points). For each of the following two arguments, please (a) translate the argument into logical notation, using *restricted* quantifiers and the provided symbols, and then (b) if the translated argument is invalid in QKυ, prove its invalidity by the tableau method, but if it is valid, prove by natural deduction its validity *in a weakest normal extension of QK in which it is valid*. Each argument is worth 1.5 point (0.5 for the translation and 1.0 for the proof).

1. The first argument in Problem 2 of Problem Set #8.
2. Possibly, there is a perfect being (i.e., a being that has all perfections essentially). Necessarily, any being that has all perfections exists necessarily. Necessarily, every perfection necessarily both exists and is a perfection. Therefore: necessarily, there is a perfect being. (*Px*: *x* is a perfection; *Hxy*: *x* has *y*.)

PROBLEM SET #10
Due: Wednesday 10 April at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) translations, semantic tableaux, and natural deduction for identity and (b) the Fitting et al. reading.

Problem 2 (3points). For each of the following two arguments, please (a) translate the argument into logical notation, using the provided symbols, and then (b) if the translated argument is invalid in QIK, prove its invalidity by the tableau method, but if it is valid, prove by natural deduction its validity. Each argument is worth 1.5 point (0.5 for the translation and 1.0 for the proof).

1. Necessarily, if there is a perfect being, then it is necessary that there be only one perfect being. Therefore: necessarily, every perfect being is such that, necessarily, it is the only perfect being. (*Px*: *x* is a perfect being.)
2. Necessarily, there is exactly one thing in my right hand. Necessarily, there is exactly one thing in my left hand. Possibly, nothing is in both my hands. Therefore: possibly, there are exactly two things in my hands. (*Rx: x* is in my right hand; *Lx*: *x* is in my left hand.)

PROBLEM SET #11
Due: Wednesday 17 April at 11:00am in class

Problem 1 (1point). In-class quiz on (a) non-rigid designators and de re modality, (b) translations, syntax, and semantics for predicate abstraction, and (c) the Thomason et al. reading.

Problem 2 (3points). Please (a) use the provided symbols and predicate abstraction to translate the following sentence into logical notation in 16 ways such that no two translated sentences are logically equivalent to each other, and then (b) find the truth value of each translated sentence at each world of the frame in Problem 2 of Problem Set #2 by using the denotation function in the table below:

The number of protons in the world is necessarily not equal to the number of quarks in the smallest star system of our galaxy but is possibly smaller than the number of stars in our galaxy. (*Sxy*: *x* is smaller than *y*; α: the number of protons in the world; β: the number of quarks in the smallest star system of our galaxy; γ: the number of stars in our galaxy.)

World	w_0	w_1	w_2	w_3
Denotation of α	10^{258}	10^{653}	3	10^{965}
Denotation of β	10^{653}	10^{258}	6	10^{332}
Denotation of γ	10^{134}	—	6	10^{965}

PROBLEM SET #12
Due: Wednesday 24 April at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) translations, semantic tableaux, and natural deduction for predicate abstraction and (b) the Fitting et al. and Plantinga readings.

Problem 2 (3 points). For each of the following two arguments, please (a) translate the argument into logical notation, using the provided symbols and predicate abstraction, and then (b) if the translated argument is invalid in λQIKυ, prove its invalidity by the tableau method, but if it is valid, prove by natural deduction its validity *in a weakest normal extension of λQIK in which it is valid*. Each argument is worth 1.5 point (0.5 for the translation and 1.0 for the proof).

1. The mereological sum of the two most famous Greek philosophers has the property of being such that, necessarily, everyone who admires the greatest admirer of Plato is distinct from it. The greatest admirer of Aristotle has the property of possibly admiring the greatest admirer of Plato. Therefore: possibly, the mereological sum of the two most famous Greek philosophers is distinct from the greatest admirer of Aristotle. (*Axy*: *x* admires *y*; μ: the mereological sum of the two most famous Greek philosophers; π: the greatest admirer of Plato; α: the greatest admirer of Aristotle.)
2. Possibly, the greatest mathematician has the property of necessarily-being-admired-by some philosopher. Necessarily, the greatest physicist has the property of possibly-admiring every philosopher. Necessarily, the greatest physicist is the greatest mathematician. Therefore: possibly, the greatest physicist has the property of possibly-both-admiring-and-being-admired-by some philosopher. (*Px*: *x* is a philosopher; *Axy*: *x* admires *y*; π: the greatest physicist; μ: the greatest mathematician.)

PROBLEM SET #13
Due: Wednesday 1 May at 11:00am in class

Problem 1 (1 point). In-class quiz on (a) definite descriptions and (b) the required readings for Part III.C of the course (Garson, Quine).

Problem 2 (3 points). For each of the following two arguments, please (a) translate the argument into logical notation, using the provided symbols, predicate abstraction, and definite descriptions, and then (b) prove by natural deduction the validity of the translated argument by using assumptions about the accessibility relation that are as weak as possible. Each argument is worth 1.5 point (0.5 for the translation and 1.0 for the proof).

1. The necessary rescuer of every rescuer of Lois Lane has the property of not necessarily rescuing Lois Lane. It follows that Lois Lane has the property of not necessarily rescuing herself. (*Rxy*: *x* rescues *y*; *l*: Lois Lane.)
2. Possibly, the universal problem-solving computer has the property of being built by some admirer of the necessary builder of Deep Blue. Necessarily, every admirer of the necessary builder of Deep Blue is such that, necessarily, she admires only herself. Therefore: possibly, the universal problem-solving computer has the property of possibly-being-built-by the necessary builder of Deep Blue. (*Px*: *x* is a problem; *Cx*: *x* is a computer; *Sxy*: *x* can solve *y*; *Bxy*: *x* builds *y*; *Axy*: *x* admires *y*; *d*: Deep Blue.)