# New Foundations for Imperative Logic: Pure Imperative Inference

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Imperatives cannot be *true*, but they can be *obeyed* or *binding*: 'Surrender!' is obeyed if you surrender and is binding if you have a *reason* to surrender. A pure *declarative* argument — whose premisses and conclusion are declaratives — is *valid* exactly if, necessarily, its conclusion is true if the conjunction of its premisses is true; similarly, I suggest, a pure *imperative* argument — whose premisses and conclusion are imperatives — is *obedience-valid* (alternatively: *bindingness-valid*) exactly if, necessarily, its conclusion is obeyed (alternatively: binding) if the conjunction of its premisses is. I argue that there are two kinds of bindingness, and that a vacillation between two corresponding variants of bindingness-validity largely explains conflicting intuitions concerning the validity of some pure imperative arguments. I prove that for each of those two variants of bindingness-validity there is an equivalent variant of obedience-validity. Finally, I address alternative accounts of pure imperative inference.

## 1. Introduction

You are given an exam that consists of six questions, numbered from 1 to 6. The instructions are as follows:

(A) Answer exactly three out of the six questions

(B) Do not answer both questions 3 and 5

(C) Answer at least one even-numbered question

After staring at the text of the instructions for a while, you exclaim: 'Wait a moment! The third instruction is redundant: it follows from the first two. If I obey the second instruction, I will answer at most two out of the three odd-numbered questions; so if I further obey the first instruction and thus I answer three questions in total, I will answer at least one even-numbered question, and thus I will automatically obey the third instruction as well.'

Your reasoning seems indeed to establish that the third instruction follows from the first two. But what exactly is it for an instruction to

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follow from other instructions? More generally, if we call *prescriptions* the entities that imperative sentences typically express (i.e. not only instructions, but also commands, requests, suggestions, etc.), what exactly is it for a prescription to follow from one or more (other) prescriptions? Equivalently, what exactly is it for a *pure imperative argument* — namely an argument[1] whose premisses and conclusion are prescriptions — to be *valid*? The development of a satisfactory answer to this question is the main object of the present paper. This question is of central importance for *imperative logic* (the proper logic of prescriptions and, derivatively, of imperative sentences): similarly to the way in which the standard definition of validity for pure declarative arguments — namely arguments whose premisses and conclusions are propositions — is the cornerstone of 'declarative' (or 'assertoric') logic, a satisfactory definition of validity for pure imperative arguments should be the cornerstone of imperative logic.

A natural approach to the above question is to define the validity of pure imperative arguments by analogy with the validity of pure declarative arguments. A pure declarative argument is valid — in other words, the conjunction of its premisses *entails* its conclusion — exactly if it 'transmits' truth from its premisses to its conclusion (more precisely: exactly if, necessarily, its conclusion is true if the conjunction of its premisses is true).[2] Similarly, the idea is to say that a pure imperative argument is valid exactly if it 'transmits' some appropriate

[1] I define an *argument* as an ordered pair whose first coordinate is a non-empty set of propositions and/or prescriptions (the *premisses* of the argument) and whose second coordinate is a proposition or a prescription (the *conclusion* of the argument). I call an argument *declarative* exactly if its conclusion is a proposition, and *imperative* exactly if its conclusion is a prescription. I call an argument *pure* exactly if its premisses and its conclusion are either all propositions or all prescriptions, and *mixed* otherwise. I call an argument *mixed-premiss* exactly if its premisses include both a proposition and a prescription. (So every mixed-premiss argument is mixed.) I use 'the argument from $I$ to $I'$' as shorthand for 'the argument whose premiss is $I$ and whose conclusion is $I'$'. If a sentence $Q$ expresses a prescription $I$ and a sentence $Q'$ expresses a prescription $I'$, I say that the sentence $\ulcorner Q;$ so $Q'\urcorner$ *expresses* the argument from $I$ to $I'$.

[2] This is a definition of *semantic* validity; in this paper I do not deal with *syntactic* validity. Given that a declarative sentence (like 'you will open the door') can express a prescription, and that an imperative sentence (like 'marry in haste and repent at leisure') can express a proposition, I take imperative logic to deal primarily with prescriptions and only secondarily with imperative sentences (and similarly I take declarative logic to deal primarily with propositions and only secondarily with declarative sentences). So I take the question of how to define semantic validity to be primary and the question of how to define syntactic validity (so as to ensure soundness and completeness) to be secondary. I plan to address the latter question in a sequel to this paper.

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property from its premisses to its conclusion (more precisely: exactly if, necessarily, its conclusion has the property if the conjunction of its premisses has it). The appropriate property cannot be *truth*, but it can be (1) *obedience* or (2) *bindingness*: it makes no sense to say that a prescription is true (or false), but it makes sense to say that a prescription is obeyed or that it is binding. For example, the prescription I (typically) express by (addressing to you the imperative sentence) 'kiss me' is obeyed if you kiss me and is binding if you have a *reason* to kiss me. Say, then, that a pure imperative argument is (1) *obedience-valid* exactly if, necessarily, its conclusion is obeyed if the conjunction of its premisses is obeyed, and is (2) *bindingness-valid* exactly if, necessarily, its conclusion is binding if the conjunction of its premisses is binding. These two definitions correspond to earlier proposals in the literature (see Appendix B). But to make the definitions precise and informative, more needs to be said on what it is for a prescription to be obeyed or to be binding.

Consider first bindingness. Say that a prescription is *binding* if it is supported by a reason and is *non-binding* otherwise. For example, if there is a reason for you to help your father but there is no reason for you to kill him, then the prescription expressed by 'help your father' is binding but the prescription expressed by 'kill your father' is non-binding. (Note that a reason or a binding prescription need not be associated with an obligation; if one objects to my use of the term 'binding' because the term suggests the existence of an obligation, one is welcome to use an alternative term instead.[3]) The distinction between binding and non-binding prescriptions may be intuitively appealing, but how can one use this distinction to decide whether any specific pure imperative argument is bindingness-valid? For example, is the argument from 'kiss me and hug me' to 'hug me' bindingness-valid? In other words, is it necessary that, if there is a reason for you to kiss and hug me, then there is a reason for you to hug me? Such questions, to my knowledge, are not answered in the literature on 'imperative inference'. But they need to be answered if bindingness-validity is to become a usable concept.

Consider next obedience. Is the (conditional) prescription expressed by 'if you meet her, warn her' obeyed if you avoid meeting her because you want to avoid warning her? (Cf. Hamblin 1987, p. 85.)

[3] See Vranas 2008, pp. 552–3, n. 7 for a list of alternative terms that have been used in the literature. I chose the term 'binding' partly because reasons *are* associated with obligations in the cases in which I am primarily interested; see Sect. 3.4.

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On the one hand, the prescription is not *violated*, as it would be if you met her without warning her. But on the other hand, the prescription is not *satisfied* either, as it would be if you met her and warned her; the prescription is rather *avoided* (i.e. neither satisfied nor violated). Should obedience be understood as non-violation (i.e. satisfaction *or* avoidance) or as satisfaction? To eliminate this ambiguity, and without claiming to capture everyday usage, I define obedience as non-violation, and thus I distinguish obedience from satisfaction for conditional prescriptions. For *unconditional* prescriptions, by contrast, namely those prescriptions that cannot be avoided (e.g. 'warn her'), obedience — i.e. satisfaction or avoidance — amounts to satisfaction. Given my definition of obedience (and contrary to what I suggested about bindingness-validity), it is clear how to decide whether any specific pure imperative argument is obedience-valid. For example, the argument from 'if you love me, kiss me and hug me' to 'if you love me, hug me' is obedience-valid: necessarily, if its premiss is obeyed (i.e. if you love, kiss, and hug me, or you do not love me), then its conclusion is also obeyed (i.e. you love and hug me, or you do not love me).

My terminological decision to define obedience as non-violation (rather than as satisfaction) does not settle the substantive issue of whether the validity of pure imperative arguments should be understood as obedience-validity or as — what may by analogy be called — *satisfaction-validity*. (To be explicit: a pure imperative argument is satisfaction-valid exactly if, necessarily, its conclusion is satisfied if the conjunction of its premisses is satisfied.) This substantive issue (couched in different terminology) has been approached in the literature on imperative inference by appealing to intuitions concerning the validity of specific (kinds of) pure imperative arguments. A major problem with this approach is that the intuitions of different people often conflict. For example, some people take arguments like the one from 'if it rains, close the window' to 'if it rains and thunders, close the window' to be valid, whereas other people take such arguments to be invalid (see Sect. 5.1). (It can be shown that such arguments are obedience-valid but satisfaction-invalid.) Moreover, since we do not know (yet) whether such — or other — arguments are bindingness-valid, a mere appeal to intuitions does not even address the (second) substantive issue of whether the validity of pure imperative arguments should be understood as bindingness-validity (rather than either as obedience-validity or as satisfaction-validity).

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These issues cry out for a more principled approach, an approach that goes beyond a mere appeal to intuitions.

In this paper I make five main contributions to the literature on imperative inference. (1) I propose (in Sect. 2) a principled way to define *pure imperative validity* — that is, the validity of pure imperative arguments. My starting point is the desire for a *useful* definition; I argue that this desire leads naturally to defining pure imperative validity as something akin to bindingness-validity. (2) I distinguish (in Sect. 3) between two kinds of bindingness, *strong* and *weak*, and between two corresponding kinds of pure imperative validity. (3) I prove (in Appendix A) that for each of those two kinds of pure imperative validity there is an equivalent variant (which I specify in Sect. 4) of obedience-validity. These equivalences enable one to decide whether specific pure imperative arguments are valid. (4) I examine (in Sect. 5) specific (kinds of) pure imperative arguments, and I argue that a vacillation between the two kinds of pure imperative validity largely explains conflicting intuitions concerning the validity of some of those arguments. (5) I argue (in Appendix B) that my definition of pure imperative validity is preferable to other definitions proposed in the literature. Taken together, these five contributions establish new foundations for pure imperative inference.

Before I begin, some remarks are in order. First, some philosophers — notably Bernard Williams (1963) — have objected to the very possibility of imperative inference, and thus might view my main project in this paper with suspicion. I have replied to their objections in another paper (Vranas 2010); here let me just say that the example with the exam instructions at the beginning of this section provides some evidence that imperative inference is possible. Second, the scope of this paper excludes *mixed-premiss* imperative arguments, namely arguments whose conclusion is a prescription and whose premisses include both a prescription and a proposition (e.g. the argument from the prescription expressed by 'if you love him, marry him' and the proposition expressed by 'you love him' to the prescription expressed by 'marry him'); I address such arguments in a sequel to this paper. Third, this paper is a sequel to another paper (Vranas 2008) but does not presuppose any familiarity with that paper. For the moment I need only the following definitions from that paper. A *prescription* is an ordered pair of logically incompatible propositions, namely the *satisfaction proposition* (the first coordinate of the pair) and the *violation proposition* (the second coordinate of the pair) of the

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prescription.[4] The disjunction of those two propositions is the *context* of the prescription; the negation of the context is the *avoidance proposition* of the prescription. A prescription is *unconditional* exactly if its avoidance proposition is impossible (equivalently, its violation proposition is the negation of its satisfaction proposition), and is *conditional* exactly if it is not unconditional. To these definitions I add: the negation of the violation proposition of a prescription is the *obedience proposition* of the prescription. (Later on I also use my definition of the *conjunction* of prescriptions; see Sect. 4.2.)

## 2. Pure imperative validity

A typical reason for adducing a valid pure *declarative* argument is to convince people that they should *believe* its conclusion. Similarly, I submit, a typical reason for adducing a valid pure *imperative* argument would be to convince people that they should *act* according to its conclusion. (The former 'should' is epistemic; the latter is practical.) This suggests that any *useful* definition of pure imperative validity will have the following consequence: necessarily, if a pure imperative argument is valid and one should act according to its premisses, then one should (also) act according to its conclusion. This suggestion, however, will turn out to be not quite right. Several complications need to be addressed; some of them arise because saying that one should act according to a prescription — or that a prescription is binding — is multiply ambiguous.

A first complication concerns the distinction between *pro tanto* (i.e. prima facie) and *all-things-considered* bindingness (or 'should'). A prescription is pro tanto binding exactly if it is supported by *some* reason, and is all-things-considered binding exactly if it is supported by the *balance* of (all) reasons.[5] For example, if there is a reason for

[4] This definition of a prescription is motivated by the idea that there is a one-to-one correspondence between all prescriptions and all ordered pairs of incompatible propositions: (1) to each prescription corresponds the pair whose first coordinate is the proposition that the prescription is satisfied and whose second coordinate is the proposition that the prescription is violated, and (2) if *S* and *V* are declarative sentences that express incompatible propositions, then the sentence ⌜if it is the case that *S* or *V*, let it be the case that *S*⌝ expresses a prescription which is satisfied exactly if *S* is true and is violated exactly if *V* is true (see Vranas 2008, pp. 532–3 for details).

[5] I understand reasons as facts (see Sect. 3.1, where I also explain what it is for a reason to *support* a prescription), and I understand the balance of reasons as the conjunction of the facts that are reasons. I do not assume that the balance of reasons is always a reason. But the balance of reasons *is* a reason when it supports a prescription (see n. 16), so

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you to help me (I need your help) but there is also a stronger reason for you not to help me (someone else needs your help more urgently) so that the balance of reasons does not favour your helping me, then the prescription expressed by 'help me' is pro tanto but not all-things-considered binding. This distinction suggests that one would like a definition of pure imperative validity having the following two consequences:

(D1) Necessarily, if a pure imperative argument is valid and the conjunction of its premisses is *pro tanto* binding, then its conclusion is *pro tanto* binding

(D2) Necessarily, if a pure imperative argument is valid and the conjunction of its premisses is *all-things-considered* binding, then its conclusion is *all-things-considered* binding

A definition of pure imperative validity that had (D1) but did not have (D2) as a consequence might still be somewhat useful, and so might be a definition that had (D2) but did not have (D1) as a consequence; but a definition that had *both* (D1) and (D2) as consequences (as the definition that I will end up defending does) would be *more* useful, and thus would be ceteris paribus preferable.

(Throughout this paper I understand bindingness simpliciter as pro tanto bindingness. So a definition of pure imperative validity as bindingness-validity has (D1) but does not have (D2) as a consequence, and is thus different from — although it will turn out to be akin to — the definition that I will end up defending.)

A second complication arises from the possibility of distinguishing *moral* reasons from reasons of other kinds (*legal*, *prudential*, *epistemic*, etc.). A prescription is (1) pro tanto *morally* binding exactly if it is supported by some moral reason, is (2) all-*moral*-things-considered binding exactly if it is supported by the balance of all moral reasons, and is (3) all-things-considered *morally* binding exactly if it is supported by the balance of all reasons and this balance is a moral reason. (Similarly for *legally*, *prudentially*, *epistemically*, etc.) These

all-thing-considered bindingness (namely support by the balance of reasons) guarantees pro tanto bindingness (namely support by some reason). (To see how the conjunction of the facts that are reasons can itself be a reason, compare: the intersection of the members of the set $\{A, B, A \cap B\}$ is itself a member of the set, namely $A \cap B$.) If there are 'exclusionary' — more generally, 'second-order' — reasons (Raz 1975a, pp. 35–48, 1975b; cf. Clarke 1977), I understand the balance of reasons as including them.

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distinctions suggest that one would like a definition of pure imperative validity having the following consequences:

(D3) Necessarily, if a pure imperative argument is valid and the conjunction of its premisses is pro tanto *morally* binding, then its conclusion is pro tanto *morally* binding

(D4) Necessarily, if a pure imperative argument is valid and the conjunction of its premisses is all-*moral*-things-considered binding, then its conclusion is all-*moral*-things-considered binding

(D5) Necessarily, if a pure imperative argument is valid and the conjunction of its premisses is all-things-considered *morally* binding, then its conclusion is all-things-considered *morally* binding

(Similarly for *legally*, *prudentially*, *epistemically*, etc.) It turns out that (D5) is no *new* desideratum: it follows from (D2).[6] By contrast, as far as I can see, (D4) does not follow from (D2) and (D3) does not follow from (D1). It might be argued, however, that one need not care about (D3) or (D4), because it is not the business of *logic* to respond to the distinctions between moral and other reasons. Still, the fact remains that a definition of pure imperative validity that had not only (D1) and (D2) but also (D3) and (D4) as consequences (as the definition that I will end up defending does) would be *more* useful than a definition that had (D1) and (D2) but did not have (D3) and (D4) as consequences, and thus would be ceteris paribus preferable.

Before I address further complications, I want to propose the following provisional — and to my knowledge novel — definition of pure imperative validity:

DEFINITION 1: A pure imperative argument is *valid* exactly if, necessarily,[7] every reason that supports the conjunction of the

[6] Indeed: necessarily, if a pure imperative argument is valid and the conjunction of its premisses is all-things-considered morally binding, then the conjunction of its premisses is all-things-considered binding and the balance of reasons is a moral reason, and then — given (D2) — the conclusion of the argument is all-things-considered binding and the balance of reasons is a moral reason, so the conclusion is all-things-considered morally binding.

[7] Necessity, validity, and entailment are understood *logically* throughout this paper, but could also be understood *metaphysically*: a distinction between logical and metaphysical validity can be made not only for pure declarative arguments (e.g. 'Alex is a human being; so Alex is not a credit card' is arguably metaphysically but not logically valid), but also for pure

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premisses of the argument also supports the conclusion of the argument[8]

Although in a sense this is the definition that I will end up defending, the definition is provisional because — due to a further ambiguity — later on (in Sect. 3.4) I will 'split' it into two definitions. Nevertheless, I wanted to propose Definition 1 at this stage mainly in order to argue that it has (D1)–(D4) as consequences. (1) Concerning (D1): necessarily, if a pure imperative argument is valid and the conjunction of its premisses is pro tanto binding, then the conjunction of its premisses is supported by some reason, and then — given Definition 1 — that reason also supports the conclusion of the argument, so the conclusion is pro tanto binding. (2) Concerning (D2): necessarily, if a pure imperative argument is valid and the conjunction of its premisses is all-things-considered binding, then the conjunction of its premisses is supported by the balance of reasons (which is thus a reason), and then — given Definition 1 — the balance of reasons also supports the conclusion of the argument, so the conclusion is all-things-considered binding. (3) Concerning (D3): necessarily, if a pure imperative argument is valid and the conjunction of its premisses is pro tanto morally binding, then the conjunction of its premisses is supported by some moral reason, and then — given Definition 1 — that moral reason also supports the conclusion of the argument, so the conclusion is pro tanto morally binding.[9] (4) Similarly concerning (D4), and also

imperative arguments (e.g. 'destroy a credit card; so destroy something that is not a human being' is arguably metaphysically but not logically valid).

[8] Whether a fact supports a prescription depends in general on further facts; e.g. whether the fact that you have promised to marry him supports the prescription 'marry him' depends, *inter alia*, on whether he is already married. This 'context-dependence' of the relation of support is implicitly incorporated in Definition 1: the claim that, necessarily, every reason that supports a prescription $I$ also supports a prescription $I'$ amounts to the claim that, given any possible world, every reason that *in that world* supports $I$ also *in that world* supports $I'$ (and specifying a world fully specifies the context). (Talking about everything that is a reason in a given world is no more problematic than talking about everything that is a fact in a given world: as I explain in Sect. 3.1, I take reasons to be facts.) Moreover, a fact can support a prescription at one time but not at another (e.g. the fact that at 7 a.m. I promise to meet you at 9 a.m. can support at 8 a.m. but not at 6 a.m. the prescription you express by 'meet me at 9 a.m.'), so in Definition 1 'supports' is understood as 'supports at (time) $t$' and the definiens is understood as including (after 'necessarily') a universal quantification over (times) $t$; I ignore this complication in the sequel. Note also that Definition 1 (like all definitions that follow), being a definition, is understood as prefixed with 'necessarily'.

[9] A — comparative (see Sect. 3.1) — moral reason can be understood as (1) a *moral* fact that supports some prescription or other, or as (2) a fact that *morally* supports some prescription or other. On either understanding, and no matter how one distinguishes moral from

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concerning the 'legal' etc. variants of (D3) and (D4). I conclude that Definition 1 satisfies all desiderata so far.

What makes the above derivations work, and what constitutes the novelty of Definition 1, is basically the fact that, if a pure imperative argument is valid (according to Definition 1) and a reason supports the conjunction of its premisses, then the *same* reason supports its conclusion. By contrast, if a pure imperative argument is *bindingness*-valid (see Sect. 1) and a reason supports the conjunction of its premisses, then *some* — not necessarily the *same* — reason supports its conclusion. But can the very same reason support two distinct prescriptions? It can: under appropriate circumstances, the fact that you have (freely) promised to tutor each of my children (is a reason that) supports both the prescription expressed by 'tutor my daughter' and the prescription expressed by 'tutor my son'. Note also that the pure imperative argument from $I$ to $I'$ (where $I$ and $I'$ are any prescriptions) is *bindingness*-valid exactly if the pure *declarative* argument from '$I$ is binding' to '$I'$ is binding' is valid; Definition 1, by contrast, does not in any obvious way reduce the validity of a pure imperative argument to the validity of a pure declarative argument.[10]

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non-moral facts or moral from non-moral support, my derivation of (D3) from Definition 1 goes through (because on either understanding a moral reason is a — comparative — reason, namely a fact that — morally or non-morally — supports some prescription or other). But on the latter understanding a moral reason may support a prescription without morally supporting it; e.g. the conjunction of the facts that you have promised to help me and that it is in your self-interest to go to the dentist is a moral reason (in the sense that it morally supports 'help me') that supports the prescription expressed by 'go to the dentist' without morally supporting it. So proponents of the latter understanding of moral reasons might contest my definition of pro tanto moral bindingness as support by some *moral* reason (e.g. they might claim that in the above example 'go to the dentist' is supported by some moral reason but is not pro tanto morally binding), and might propose defining pro tanto moral bindingness as *moral* support by some reason. On this alternative definition my derivation of (D3) from Definition 1 does not go through, but Definition 1 might still be useful because it has (D1) and (D2) as consequences. (Proponents of the former understanding of moral reasons might claim that in the above example the conjunction of the two facts is not a moral reason because it is not a moral fact, but 'help me' — unlike 'go to the dentist' — is pro tanto morally binding because it is supported by the moral fact that you have promised — or that you have a moral obligation — to help me.)

[10] It turns out, however, that *if* a certain assumption holds, then a variant of Definition 1 with respect to *complete* reasons is equivalent to a corresponding variant of the definition of bindingness-validity. (Say that a reason $R$ is *complete* exactly if, necessarily, it *essentially* supports every prescription that it supports; i.e. necessarily, if $R$ supports $I$, then $R$ could not have existed without supporting $I$. An example is arguably the fact that I have promised to meet you and I can meet you and you have not released me from my promise and … ; cf. Raz 1975a, pp. 22–5.) More specifically, say that a pure imperative argument (whose premisses have the prescription $I$ as their conjunction and whose conclusion is the prescription $I'$) is *valid**

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Objecting to Definition 1, one might claim that the definition has the unpalatable consequence that the argument from 'travel from London to Paris' to 'travel *by train* from London to Paris' is valid: every reason for you to travel from London to Paris is a (usually not conclusive) reason for you to take the train from London to Paris. In reply I deny that, necessarily, every reason that supports the premiss of the above argument also supports the conclusion of the argument: under appropriate circumstances, the fact that you have (freely) promised to travel *by plane but not by train* from London to Paris (is a reason that) supports 'travel from London to Paris' but does not support 'travel by train from London to Paris'. So Definition 1 does not have the consequence that the above argument is valid, and the objection fails.[11]

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exactly if, (B1) necessarily, every *complete* reason that supports $I$ also supports $I'$, and is *bindingness-valid** exactly if, (B2) necessarily, if some *complete* reason supports $I$ then some *complete* reason supports $I'$. Clearly, (B2) follows from (B1): see my derivation of (D1) from Definition 1. Conversely, (B1) follows from the conjunction of (B2) with the following assumption: (A1) necessarily, every complete reason could have been the *only* complete reason. (Cf. Kelsen 1979, p. 186, 1991, p. 233. One might object that the fact that I have promised to help you exactly if I have some other reason to help you (cf. Dancy 2004b, p. 19) could not have been the only reason; I reply that the above fact is not a complete reason.) To see that (B1) follows from (B2)&(A1), take any possible world $W$ and any complete reason $R$ that in $W$ supports $I$. By (A1), there is a possible world $W'$ in which $R$ is the only complete reason (and thus exists). Since $R$ in $W$ supports $I$ and $R$ is a complete reason, $R$ essentially supports $I$. So $R$ in $W'$ supports $I$, and by (B2) some complete reason $R'$ in $W'$ supports $I'$. But $R$ is the only complete reason in $W'$, so $R' = R$ and $R$ in $W'$ supports $I'$. Since $R$ is a complete reason, $R$ essentially supports $I'$, so $R$ in $W$ supports $I'$, and (B1) holds. (It can also be shown that, if (A1) holds, then bindingness-validity* is equivalent to *all-things-considered* bindingness-validity*, defined in terms of: (B3) necessarily, if the balance of all *complete* reasons supports $I$, then it also supports $I'$. Similarly, the variants of (D1) and of (D2) with respect to complete reasons are equivalent if (A1) holds.)

[11] Against Definition 1, and against reasons- or bindingness-based definitions of pure imperative validity in general, one might raise an objection resting on a distinction (cf. Vranas 2008, p. 554, n. 15) between *personal* prescriptions, commonly called *directives* (e.g. 'Lou, turn on the light'), and *impersonal* prescriptions, commonly called *fiats* (e.g. 'let there be light'). The objection relies on the premiss that reasons cannot support impersonal prescriptions (they can only support personal ones); e.g. it makes no sense to say that a reason supports the impersonal prescription expressed by 'let the volcano erupt' (cf. Scanlon 1998, p. 18). From this premiss the objection infers that every *impersonal* pure imperative argument — namely every argument whose premisses and conclusion are impersonal prescriptions, e.g. the argument from 'let it rain' to 'let it snow' — is trivially valid according to Definition 1: necessarily, no reason supports the conjunction of the premisses of such an argument (since that conjunction is an impersonal prescription), so it is trivially true that, necessarily, every reason that supports the conjunction of the premisses of such an argument also supports the conclusion of the argument. In reply I contest the premiss of the objection: I submit that reasons *can* support impersonal prescriptions. Suppose that a huge earthquake in a populous city would be averted if a volcano were to erupt. The fact that the volcano's eruption would prevent many deaths is a

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One might be worried by the fact that Definition 1 looks very different from the standard definition of pure declarative validity. It turns out, however, that the two definitions can be put into a common format. Say that a fact *supports* a proposition exactly if, necessarily, if the fact exists (i.e. is a fact) then the proposition is true. (If one accepts what Rodriguez-Pereyra (2006, p. 958) calls a 'traditional definition of truthmaking', then one can equivalently say that a fact supports a proposition exactly if the fact is a *truthmaker* for the proposition.) Then, as I explain in a note, a pure declarative (like a pure imperative) argument is valid exactly if, necessarily, every fact that supports the conjunction of the premisses of the argument also supports the conclusion of the argument.[12] But then Definition 1 and the standard definition of pure declarative validity are in a sense not very different after all.[13]

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consideration that counts in favour of the volcano's erupting; so if 'a reason for something [is] a consideration that counts in favor of it' (Scanlon 1998, p. 17), there is a reason — in Scanlon's (1998, p. 219) terminology, a *personal* reason — for the volcano to erupt. This is not a reason that the volcano *has*, although I can grant that it is a reason that *people* have to make the volcano erupt (if they can). Moreover, if *impersonal* reasons — namely 'reasons that are not tied to the well-being, claims, or status of individuals in any particular position' (Scanlon 1998, p. 219) — can exist, they can also support impersonal prescriptions; an example would be the fact that the volcano's eruption would prevent the destruction of natural beauty. I conclude that reasons can support impersonal prescriptions, and the objection fails. It is important to note, however, that those who insist that reasons cannot support impersonal prescriptions can just restrict Definition 1 to arguments whose premisses and conclusions are personal prescriptions.

[12] The result about pure imperative arguments is easy given that, necessarily, every reason is a fact and every fact that supports a prescription is a reason (see Sect. 3.1). To see that the result about pure declarative arguments holds, consider a pure declarative argument whose premisses have the proposition $P$ as their conjunction and whose conclusion is the proposition $P'$. Suppose that, (1) necessarily, every fact that supports $P$ also supports $P'$. To show that (2) $P$ entails $P'$, take any possible world $W$ in which $P$ is true (if no such world exists, then (2) holds trivially). Then in $W$ the fact that $P$ is true supports $P$ and thus (by (1)) also supports $P'$; so $P'$ is also true in $W$, and (2) holds. Conversely, suppose that (2) holds. To show that (1) holds, take any possible world $W$ in which some fact $f$ supports $P$ (if no such world exists, then (1) holds trivially). Then, necessarily, if $f$ is a fact then $P$ is true. But then, necessarily, if $f$ is a fact then $P'$ is also true (since $P$ entails $P'$); so $f$ also supports $P'$ in $W$, and (1) holds.

[13] One might wonder whether the following analogue of Definition 1 for pure declarative arguments succeeds for (i.e. is equivalent to the standard definition of) pure declarative validity: a pure declarative argument is valid exactly if, necessarily, every reason for a person to believe the conjunction of the premisses of the argument is a reason for the person to believe the conclusion of the argument. The answer is arguably negative (see n. 50), but later on (in Sect. 4.2) I explain that the following analogue of Definition 1 does succeed: a pure declarative argument is valid exactly if, necessarily, every reason that supports the prescription 'let it be the case that the conjunction of the premisses is true' also supports the prescription 'let it be the case that the conclusion is true' (see n. 36 for a more precise formulation).

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At this stage Definition 1 looks quite promising. But how, on the basis of this definition, can one decide whether any specific pure imperative argument is valid? To answer this question, one needs to examine what it is for a reason to support a prescription. I turn next to such an examination, which will raise further complications.

## 3. Strong and weak bindingness

### *3.1 Reasons and support*

What is a (normative or justificatory, not a motivating or explanatory) reason? Reasons are in a sense like parents: someone cannot be a parent without being a parent *of* someone, and similarly something cannot be a reason without being a reason *for* something (or *against* something, but for my purposes in this paper I hardly ever need to talk again about reasons against). Given the 'commonplace' (cf. Broome 2004, p. 41; Hieronymi 2005, p. 437; Parfit 2001, p. 18) that a reason for something is a consideration that counts in favour of it, it is natural to say that a reason is a consideration that counts in favour of something. I take the 'considerations' in question to be *facts* (cf. B. Gert 2002, p. 284, 2004, p. 103; Pryor 2007; Raz 1975a, pp. 16–18; Schroeder 2008, pp. 63–4; Skorupski 1997, pp. 345–6; Smith 2002, pp. 113–16) rather than for example propositions or beliefs, although nothing substantive in this paper hangs on this choice. I thus take a reason to be a fact that counts in favour of — in short, that *favours* (Dancy 2004b, p. 29) — something. Arguably, things of more than one kind can be favoured by reasons: there can be reasons for actions, reasons for beliefs, and so on. Nevertheless, rather than talking about reasons for an action, one can talk about reasons for the proposition that the action is performed; similarly concerning reasons for a belief, and so on, so I can — and I will — take the 'things' that can be favoured by reasons to be *propositions* (contrast Wedeking 1970, p. 163). I thus take the relation of being a reason for — in other words, of counting in favour of, or of favouring — to relate facts with propositions.[14] This relation is distinct from the property

[14] One might claim that, since on my approach a reason is a fact that favours some proposition, my approach blurs the distinction between (1) reasons for actions and reasons for beliefs, and also blurs the distinction between (2) pragmatic (or practical) and epistemic (or evidentiary) reasons. In reply note first that on my approach one *can* make the above distinctions as long as one can distinguish (1′) propositions to the effect that someone performs an action from propositions to the effect that someone has a belief and (2′) pragmatic from epistemic (kinds of) favouring. I grant, however, that my approach blurs distinctions (1)

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of being a reason, a property of facts; by analogy, the property of being a parent is distinct from the relation of being a parent of (cf. Raz 1975a, p. 23).

So far in this section I have been talking about *non-comparative* reasons (and favouring). In contrast to a non-comparative reason, which is a fact that (non-comparatively) favours some proposition, a *comparative* reason is a fact that (comparatively) favours some proposition *over* some other one.[15] For example, the fact that you have promised to reveal a certain secret to both of your parents *if* you reveal it to either of them is normally a reason for you to reveal the secret to both of your parents *rather than* to only one of them. In other words, the above fact normally favours the proposition that you reveal the secret to both of your parents over the proposition that you reveal it to only one of them. Equivalently, the above fact normally *favours the satisfaction over the violation proposition* of the prescription expressed by 'reveal the secret to both of your parents if you reveal it to either of them'. As it is also natural to say that the above fact normally *supports* the above prescription, I propose the following definition:

> DEFINITION 2: A (fact which is a comparative) reason *supports* a prescription exactly if it (comparatively) favours the satisfaction over the violation proposition of the prescription[16]

I assume that the relation of comparative favouring is *asymmetric*: any fact that favours a proposition $P$ over a proposition $P'$ does not also favour $P'$ over $P$. Definition 2 suggests that for the purposes of imperative logic one is primarily interested in comparative rather than

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and (2) in so far as my talk of reasons encompasses *all* reasons (including reasons for actions, reasons for beliefs, pragmatic reasons, epistemic reasons, moral reasons, legal reasons, and so on). But this blurring is intentional: I understand 'every reason' in Definition 1 as '*every* reason, no matter of what kind', and I do so partly because I want Definition 1 to apply not only to arguments like 'surrender; so surrender or fight' but also to arguments like 'believe that he surrendered; so believe that he surrendered or fought' (see Sect. 5.2.4 for discussion of an argument like the latter one and of the distinction between non-epistemic and epistemic reasons).

[15] Note that a reason might be *both* comparative and non-comparative: a fact might comparatively favour a first proposition over a second one and also non-comparatively favour a third proposition. Note also that a reason might be both moral and non-moral (cf. n. 9), and that — as I suggested in n. 14 — different kinds of favouring might exist (if they do, I understand Definition 1 as including a universal quantification over kinds of favouring).

[16] I understand Definition 2 as entailing that *only* (comparative) reasons support prescriptions; similarly for Definitions 3, 4, and 5 in the sequel. By contrast, I understand Definition 1 as *not* entailing that only pure imperative arguments are valid; e.g. pure declarative arguments can also be valid compatibly with Definition 1.

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non-comparative reasons (and favouring). One might object by arguing that comparative reasons can be reduced to non-comparative ones; for example, a fact is a reason for you to do A rather than B exactly if it is a *stronger* (non-comparative) reason for you to do A than it is for you to do B. I reply that a fact can be a reason for you to do A rather than B without being at all a (non-comparative) reason for you to do A; for example, the fact that you have promised to reveal a certain secret to both of your parents *if* you reveal it to either of them is *not* a reason for you to reveal the secret to both of your parents. Conversely, one might argue that non-comparative reasons can be reduced to comparative ones: a fact non-comparatively favours a proposition exactly if it comparatively favours the proposition over its negation. This may well be correct; if it is, then one can show (by using Definition 2 and the definition of an unconditional prescription) that a reason supports an unconditional prescription exactly if it *non*-comparatively favours the satisfaction proposition of the prescription (and this makes redundant any reference to comparative favouring if one considers only unconditional prescriptions).

Objecting to Definition 2, one might claim that a reason supports a prescription exactly if it favours the *obedience* — rather than the *satisfaction* — over the violation proposition of the prescription. This claim has the consequence that any two prescriptions that have the same violation (and thus also the same obedience) proposition are supported by the same reasons. But this consequence is unpalatable, as shown by the following example. Suppose it is a fact that, for the purpose of promoting your health, exercising and dieting is better than exercising without dieting, and exercising without dieting is better than neither exercising nor dieting. I submit that this fact supports the prescription expressed by 'if you exercise, also diet' (since exercising and dieting is better than exercising without dieting) but does not support the prescription expressed by 'if you do not diet, do not exercise either' (since exercising without dieting is better than neither exercising nor dieting). And yet the two prescriptions have the same violation proposition, namely the proposition that you exercise without dieting. Consider also another example: the fact that you have promised to flee if the volcano erupts normally supports the (personal) prescription expressed by 'if the volcano erupts, flee' but does not support the (impersonal) prescription expressed by 'if you do not flee, let it not be the case that the volcano erupts', although the two prescriptions have the same violation proposition (namely the proposition that the volcano erupts and you do not flee). So not

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every two prescriptions that have the same violation proposition are supported by the same reasons, and the above objection to Definition 2 fails.[17]

*3.2 Strong bindingness*

I turn now to the task of distinguishing two kinds of cases in which a reason supports a prescription, and thus also two kinds of bindingness (namely strong and weak). Suppose it is a fact that I have promised to help you. This fact normally favours the proposition that I help you over the proposition that I do not help you. In other words (to introduce a paraphrase), *relative* to the fact that I have promised to help you, it is normally *better* if I help you than if I do not help you. (In what follows I omit the qualifier 'normally', which is meant to exclude e.g. cases of coerced promises.) Moreover, I submit that the following *dominance* condition holds: the fact that I have promised to help you favours *every* proposition which entails that I help you over *every* different proposition which entails that I do not help you. (The propositions must be different because the fact that I have promised to help you does not favour an impossible proposition over itself, although an impossible proposition entails both that I help you and that I do not help you.[18]) This condition holds because (1) every proposition which entails that I help you also entails that I do not break any promise I have made to help you, (2) every proposition which entails that I do not help you also entails that I break any promise I have made to help you, and (3) relative to the fact that I have promised to help you it is better if I do anything which entails that I do not break my promise than if I do anything else which entails that I break it. One might object by claiming that my helping *and then*

[17] Objecting further to Definition 2, one might claim that a reason supports a prescription exactly if it favours the satisfaction proposition of the prescription over the *negation* of the satisfaction proposition. This claim has the consequence that any two prescriptions that have the same satisfaction proposition are supported by the same reasons. But this consequence is unpalatable: the fact that you have promised to help me *if* war breaks out supports the prescription expressed by 'if war breaks out, help me' but does not support the prescription expressed by 'let it be the case that war breaks out and you help me', although the two prescriptions have the same satisfaction proposition.

[18] I assume that necessarily equivalent propositions are indistinguishable with respect to the relation of comparative favouring, so *for the sake of simplicity* throughout this paper I assume that necessarily equivalent propositions are identical. Dropping the latter assumption would make some of my formulations cumbersome; e.g. rather than saying that no fact favours a proposition over itself, I would say that no fact favours a proposition $P$ over any proposition necessarily equivalent to $P$, and in the formulation of the dominance condition I would replace 'different' with 'not necessarily equivalent'.

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*hitting* you (which entails that I help you) is *not* better than my neither helping nor hitting you (which entails that I do not help you). But even if it is not better *simpliciter*, I reply, it is still better *relative* to the fact that I have promised to help you. Indeed, *as far as my promise to help you is concerned*, what matters is whether I help you or not; it does not matter whether I hit you or not.[19] (And recall that the fact that I have promised to help you can be a *comparative* reason for me to help and then hit you — *rather than* neither help nor hit you — without being a *non*-comparative reason for me to help and then hit you.) I conclude that the dominance condition holds.[20]

Similarly, I submit that the following *indifference* condition holds: the fact that I have promised to help you does not favour any proposition which entails that I help you over any other such possible proposition. Indeed, every such proposition entails that I do not break any promise I have made to help you, so no such proposition is better than any other relative to the fact that I have promised to help you. (With one kind of exception: the dominance condition entails that every possible proposition which entails that I help you is better — relative to the fact that I have promised to help you — than an impossible proposition.[21] Hence the requirement, in the

[19] One might object that my promise to help you should be understood as a promise to help you *without harming* you; but hitting you counts as harming you, so as far as my promise is concerned it does matter whether I hit you or not. In reply distinguish an unconditional promise that is kept exactly if I help you (which is how I understand in the text my promise to help you) from an unconditional promise that is kept exactly if I help you *without harming* you. I can grant that the fact that I have made the latter promise does not favour every proposition which entails that I help you over every different proposition which entails that I do not help you; but it does favour every proposition which entails that I help you *without harming* you over every different proposition which entails that I do not, so the *relevant* dominance condition does hold.

[20] One might object to the dominance condition by claiming that the fact that I have promised to help you favours the proposition that (1) I do not set off a bomb exactly if I have promised to help you over the proposition that (2) I set off a bomb exactly if I have promised to help you, so the above fact does not favour the conjunction — call it (3) — of (2) with the proposition that I help you over the conjunction — call it (4) — of (1) with the proposition that I do not help you, although (3) entails that I help you and (4) entails that I do not help you. In reply I can grant that, *if* I have promised to help you, then (1) is better than (2), but I deny that the fact that I have promised to help you favours (1) over (2): as far as my promise to help you is concerned, what matters is whether I help you, not whether I set off a bomb. What favours (1) over (2) is instead the fact that I have promised to help you *and* (e.g.) some people will die if I set off a bomb.

[21] And this seems indeed plausible: the fact that I have promised to help you arguably supports the prescription you express by 'if you help me (and you do X), help me', and thus favours the satisfaction proposition of this prescription (which entails that I help you) over the violation proposition (which is an impossible proposition).

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indifference condition, that the second proposition be *possible*.) One might object by claiming that the fact that I have promised to help you favours the proposition that I help you and I *plan* to help you over the proposition that I help you and I *do not* plan to help you — although each of these propositions entails that I help you. I reply that this amounts to the claim that the fact that I have promised to help you supports (i.e. favours the satisfaction over the violation proposition of) the prescription you express by 'if you help me, plan to help me' — and I see no reason to accept this claim. Those who say that the above claim is plausible may be confusing it with the claim (whose plausibility I can grant) that the fact that I have promised to help you favours the proposition that I help you and I plan to help you over its *negation*. I conclude that the indifference condition holds.

The above remarks motivate the following definition:

> DEFINITION 3: A (fact which is a comparative) reason *strongly* supports a prescription exactly if (1) it favours every proposition which entails the satisfaction proposition of the prescription over every different proposition which entails the violation proposition of the prescription (*dominance condition*), and (2) it does not favour any proposition which entails the satisfaction proposition of the prescription over any other such possible proposition (*satisfaction indifference condition*)

(A third condition might also be needed, namely that the prescription be *non-empty*; i.e. that its context — namely the disjunction of its satisfaction and violation propositions — be possible. This condition might be needed because the dominance and satisfaction indifference conditions trivially hold for a prescription whose satisfaction and violation propositions are both impossible, so without the condition *every* reason would strongly support an empty prescription. In the sequel I ignore empty prescriptions; i.e. whenever I — implicitly — use a quantifier ranging over prescriptions, I assume that it ranges only over non-empty prescriptions.)

In addition to *satisfaction* indifference, in some cases of strong support what may be called *violation* indifference holds. For example, since every proposition which entails that I *do not* help you also entails that I break any promise I have made to help you, no such proposition is better than any other relative to the fact that I have promised to

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help you.[22] So why does Definition 3 not require violation indifference in addition to satisfaction indifference? Because in other cases of strong support violation indifference fails. For example, suppose it is a fact that I have promised to help you and, if nevertheless I do not help you, to at least apologize. This fact, I submit, strongly supports the prescription you express by 'help me' (since in the current example, just as in examples in which I only promise to help you, I break my promise exactly if I do not help you[23]), but violation indifference fails: as far as my promise is concerned, it is better if I do not help you and I apologize than if I do not help you and I do not apologize, although in cases of both kinds I do not help you and thus the prescription you express by 'help me' is violated.[24] I conclude that there are cases of strong support in which violation indifference fails; this is why Definition 3 does not require violation indifference.[25]

[22] With one kind of exception: the dominance condition entails that an impossible proposition is better — relative to the fact that I have promised to help you — than every possible proposition which entails that I do not help you. And this seems indeed plausible: the fact that I have promised to help you arguably supports the prescription you express by '(even) if you do not help me (and you do X), help me', and thus favours the satisfaction proposition of this prescription (which is an impossible proposition) over the violation proposition (which entails that I do not help you).

[23] In the current example I assume that I have made a *single* promise (which specifies a second-best scenario), not the *two* distinct promises (1) to help you and (2) to apologize if I do not help you.

[24] In response one might try to modify the above example so as to show that in other cases of strong support *satisfaction* indifference fails. For example, suppose it is a fact that I have promised to help you and, if I do help you, to also accept your help. This fact supports the prescription you express by 'help me', but satisfaction indifference fails: as far as my promise is concerned, it is better if I help you and I accept your help than if I help you and I do not accept your help, although in cases of both kinds I help you and thus the prescription you express by 'help me' is satisfied. I reply that this is not a case of *strong* support. First, because I break my promise not (as in the previous examples) exactly if I do not help you, but rather exactly if I do not help you *or* I do not accept your help. Second, because dominance fails: as far as my promise is concerned, it is *not* better if I help you and I do not accept your help than if I do not help you and I accept your help, although the prescription you express by 'help me' is satisfied in cases of the former kind and is violated in cases of the latter kind. I conclude that there seem to be no cases of strong support in which satisfaction indifference fails; this is why Definition 3 requires satisfaction indifference.

[25] In some cases of strong support, what may be called *obedience* indifference holds. For example, since every proposition which entails that I do not both run and smile (in other words, that the prescription you express by 'if you run, do not smile' is obeyed) also entails that I do not break any promise I have made to not smile if I run, no such proposition is better than any other such possible proposition relative to the fact I have promised to not smile if I run. So why does Definition 3 not require obedience indifference (rather than just satisfaction indifference)? Because in other cases of strong support obedience indifference fails. For example, suppose it is a fact that I have promised to not smile if I run and I have also

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(Actually, as I explain at the end of Appendix A, for my purposes in this paper it does not really matter whether one includes satisfaction indifference or violation indifference — or both, or neither — in a definition of strong support: it turns out that all of these definitions correspond to the *same* useful kinds of pure imperative validity.)

*3.3 Weak bindingness*

Consider now a second kind of case in which a reason supports a prescription. Suppose it is a fact that I have promised to both help you and accept your help. It can be seen from what was said above that this fact strongly supports the prescription you express by 'help me and accept my help' but does *not* strongly support the prescription you express by 'help me'. And yet the above fact gives me an obligation to help you and thus does support (though not strongly) the prescription you express by 'help me'. Say then that the fact supports the prescription *weakly*. More generally, if a fact *strongly* supports an unconditional prescription $I^\star$ whose satisfaction proposition entails the satisfaction proposition of an unconditional prescription $I$ (e.g. the proposition that you help me and you accept my help entails the proposition that you help me), then the fact *weakly* supports $I$. Similar points can be made for *conditional* prescriptions: the fact that I have promised to both help you and accept your help *if* war breaks out gives me an obligation to help you if war breaks out and thus supports (though not strongly) the prescription you express by 'help me if war breaks out'. These remarks motivate the following definition:

> Definition 4: A (fact which is a comparative) reason *weakly* supports a prescription $I$ exactly if it strongly supports some prescription $I^\star$ whose satisfaction proposition entails the satisfaction proposition of $I$ and whose context is the same as the context of $I$

(The clause 'whose context is the same as the context of $I$' can be motivated by showing that Definition 4 also accounts for cases in which $I^\star$ and $I$ have different contexts.[26]) From Definition 4 it follows

promised to run if I do not smile. This fact strongly supports the prescription you express by 'if you run, do not smile', but obedience indifference fails: because the above fact also (strongly) supports the prescription you express by 'if you do not smile, run', it favours the proposition that I do not smile and I run over the proposition that I do not smile and I do not run, although both propositions entail that the prescription you express by 'if you run, do not smile' is obeyed. I conclude that there are cases of strong support in which obedience indifference fails; this is why Definition 3 does not require obedience indifference.

[26] The original motivation for Definition 4 is the observation: (1) if a reason $R$ strongly supports a prescription $I$ with context $C$ and violation proposition $V$, $C$ is the *same* as $C'$,

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(by letting $I^{\star}$ be the same as $I$) that a fact that strongly supports a prescription also weakly supports the prescription: all cases of strong support are cases of weak support. To have a label for the remaining cases of weak support, say that a fact *loosely* supports a prescription exactly if it supports the prescription weakly but not strongly. So no fact supports a prescription both loosely and strongly, and a fact supports a prescription weakly exactly if it supports the prescription either loosely or strongly. Note that Definition 4 does not guarantee that a fact that weakly supports a prescription also supports the prescription (i.e. favours the satisfaction over the violation proposition of the prescription). (Definition 3, by contrast, does guarantee — through the dominance condition — that a fact that strongly supports a prescription also supports the prescription.) Indeed, it can be shown that not all (possible) cases of weak support are cases of support.[27] Still, the examples that motivated Definition 4 indicate that typical cases of weak support are cases of support; for example, the fact that I have promised to both help you and accept your help favours the proposition that I help you over the proposition that I do not help you.

Objecting to my claim that typical cases of weak support are cases of support, one might argue that the fact that I have promised to post a letter does not support the prescription expressed by 'post *or burn* the letter', although it does weakly support this prescription because (1) it strongly supports the prescription expressed by 'post the letter' and (2) the proposition that I post the letter entails the proposition that I post or burn it. I defer a detailed examination of issues related to this objection until section 5.3 (where I address 'Ross's paradox'), but for

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and $V'$ entails $V$ (equivalently, the satisfaction proposition $S$ of $I$ entails $S'$), then $R$ weakly supports the prescription $I'$ with context $C'$ and violation proposition $V'$ (and satisfaction proposition $S'$). But a further observation is: (2) if $R$ strongly supports $I$, $C$ is *entailed* by $C'$, and $V'$ entails $V$, then $R$ weakly supports $I'$; e.g. the fact that I have promised to both help you and accept your help weakly supports the prescription you express by 'help me if war breaks out'. It turns out, however, that Definition 4 accounts for (2): in conjunction with Definition 3, Definition 4 entails (2). This follows from the third part of the proof of the Equivalence Theorem in Appendix A (which is the only part that does not rely on Assumption 1 of Appendix A).

[27] Given any contingent proposition $C$ and any proposition $S$ that entails $C$, it can be shown (by using Assumption 1 of Appendix A and the Equivalence Theorem of Sect. 4.1) that possibly some fact weakly supports both (1) the prescription whose satisfaction proposition is $S$ and whose violation proposition is $C\&\neg S$ and (2) the prescription whose satisfaction proposition is $C\&\neg S$ and whose violation proposition is $S$. But given the asymmetry of comparative favouring, it is impossible for a fact to support both of these prescriptions. So at least one of these prescriptions is such that possibly some fact weakly supports it without supporting it.

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the moment let me just say the following. Resistance to my claim that the fact that I have promised to post a letter is a reason for me to post or burn it (rather than neither posting nor burning it) arises mostly, I suspect, from confusing it with the claim that the fact that I have promised to post a letter is a reason for me to post or burn it *at my choice*. But the former claim, unlike the latter, does not incorporate satisfaction indifference: it is a claim of support, not a claim of *strong* support. Once this is understood, most of the resistance to my claim should vanish.

### *3.4 Strong, weak, and satisficing validity*

I said in section 2 that the definition of pure imperative validity I was proposing, namely Definition 1, was provisional because later on I would 'split' it into two definitions. The time is ripe to effect this 'split':

> Definition 1a: A pure imperative argument is *strongly valid* exactly if, necessarily, every reason that *strongly* supports the conjunction of the premisses of the argument also *strongly* supports the conclusion of the argument
>
> Definition 1b: A pure imperative argument is *weakly valid* exactly if, necessarily, every reason that *weakly* supports the conjunction of the premisses of the argument also *weakly* supports the conclusion of the argument

Say that $I$ — or $\{I\}$ — *strongly entails* $I'$ exactly if the argument from $I$ to $I'$ is strongly valid, and that $I$ *weakly entails* $I'$ exactly if the argument from $I$ to $I'$ is weakly valid. Note that strong and weak validity are different *kinds* — not different *degrees* — of validity, and similarly for entailment and for bindingness: unlike reasons, which can be stronger or weaker, validity, entailment, and bindingness — whether strong or weak — do not come in degrees.

One might propose defining two further kinds of validity. (1) Say that a pure imperative argument is *w/s valid* exactly if, necessarily, every reason that *weakly* supports the conjunction of its premisses *strongly* supports its conclusion. This kind of validity, however, violates reflexivity: the argument from $I$ to $I$ itself need not be w/s valid (because a reason can support a prescription weakly but not strongly).[28] (2) Say that a pure imperative argument is *s/w valid*

[28] One might argue that reflexivity fails: the argument from 'please go' to 'go' is invalid although its premiss is the same prescription as its conclusion, namely the unconditional

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exactly if, necessarily, every reason that *strongly* supports the conjunction of its premisses *weakly* supports its conclusion. This kind of validity, however, violates transitivity: even if the arguments from $I$ to $I'$ and from $I'$ to $I''$ are both s/w valid, the argument from $I$ to $I''$ need not be. Strong and weak validity, by contrast, are easily shown to satisfy both reflexivity and transitivity. Note that strong and weak validity might also be called *s/s* and *w/w* validity respectively. It turns out, however, that every strongly valid pure imperative argument is also weakly valid (see Sect. 4.2), so I prefer the labels 'strong' and 'weak'.[29]

One might also propose defining yet another kind of validity, based on a third kind of support. A case in which there is a reason for me to feed your cat just because I have promised to feed *either* your cat or your dog differs from a case in which there is a reason for me to feed your cat just because I have promised to feed *both* your cat and your dog: in the latter case, but not in the former, I have an *obligation* to feed your cat. The latter case corresponds to *weak* support, whereas the former corresponds to what may be called *satisficing* support. Note that my feeding your cat is *necessary* for my feeding both your cat and your dog but is *sufficient* for my feeding either your cat or your dog; more generally (and somewhat roughly), weak support corresponds to necessary conditions for strong support, whereas satisficing support corresponds to sufficient conditions for strong support.[30]

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prescription which is satisfied exactly if you go (cf. Aune 1977, pp. 176–7; Good 1986; Miller 1984, pp. 57–8), and similarly for the argument from 'I advise you to go' to 'I command you to go' (cf. Warnock 1976, pp. 296–9). I reply that the above pure imperative arguments are valid *if* each of them, for some sentence $Q$, can be expressed by $\ulcorner Q$; so $Q\urcorner$. The illusion of invalidity is due to the fact that e.g. 'I advise you to go; so I command you to go' can also express an invalid pure *declarative* argument.

[29] One might also propose saying that a pure imperative argument is *loosely valid* exactly if it is weakly but not strongly valid. This kind of validity, however, violates reflexivity: the argument from $I$ to $I$ itself is strongly and thus not loosely valid. One might further propose saying that a pure imperative argument is *l/l valid* exactly if, necessarily, every reason that *loosely* supports the conjunction of its premisses also *loosely* supports its conclusion. This definition is not useful, however, because it does not even have as a consequence the variant of (D1) (see Sect. 2) that corresponds to weak bindingness.

[30] The distinction between 'reasoning to necessary and to sufficient conditions' has been emphasized in the context of imperative logic by Hare (1969). (Cf. Kenny 1975, pp. 88–90; Raz 1978b, p. 10.) Satisficing support has some affinity with Dancy's (2004a, 2004b, pp. 21, 24) 'enticing' (as opposed to 'peremptory') reasons and with J. Gert's (2004) 'justifying' (as opposed to 'requiring') role of reasons (contrast J. Gert 2004, pp. 25–6).

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These remarks motivate the following definitions:

> DEFINITION 5: A (fact which is a comparative) reason *satisficingly* supports a prescription *I* exactly if it strongly supports some prescription $I^{\star}$ whose satisfaction proposition is entailed by the satisfaction proposition of *I* and whose context is the same as the context of *I*

> DEFINITION 1c: A pure imperative argument is *satisficingly* valid exactly if, necessarily, every reason that *satisficingly* supports the conjunction of the premisses of the argument also *satisficingly* supports the conclusion of the argument

Note that all cases of strong support are cases of satisficing support. (More generally, it can be shown that all and only cases of strong support are cases of both satisficing and weak support.) I defer a detailed examination of satisficing validity until section B.2.2 (where I examine Kenny's 'satisfactoriness-validity'), but for the moment let me just say the following. Call a prescription *obligatory (at a given time, for a given agent)* exactly if its satisfaction proposition is obligatory (at the given time, for the given agent) given its context; for example, the prescription expressed by 'let the office be open by 8 a.m.' is obligatory (at 7 a.m., for me) exactly if the proposition that the office is open by 8 a.m. is obligatory (at 7 a.m., for me).[31] In section 2 I suggested that any useful definition of pure imperative validity will have the following consequence: necessarily, if a pure imperative argument is valid and one should act according to (the conjunction of) its premisses, then one should act according to its conclusion. If one understands 'should act' in terms of obligatoriness (rather than bindingness), the desired consequence becomes:

> (D6) Necessarily, if a pure imperative argument is valid and the conjunction of its premisses is obligatory (at a given time, for a given agent), then its conclusion is obligatory (at the given time, for the given agent)

(Strictly speaking, (D6) should be replaced with a collection of claims corresponding to pro tanto, all-things-considered, moral, legal, etc.

[31] For conditional prescriptions there is a complication. Intuitively, the prescription expressed by 'let the office be open by 8 a.m. *if there is no strike*' is obligatory exactly if (1) the proposition that the office is open by 8 a.m. is obligatory given that there is no strike. But according to what I said in the text, the above prescription is obligatory exactly if (2) its satisfaction proposition, namely the proposition that the office is open by 8 a.m. *and there is no strike*, is obligatory given its context (i.e. given that there is no strike). It turns out, however, that (1) and (2) are equivalent.

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obligatoriness.) Definition 1c, however, does not have (D6) as a consequence, because satisficing support does not guarantee the existence of an appropriate obligation: as we saw, I have no obligation to feed your cat if the prescription you express by 'feed my cat' is supported (satisficingly) just by the fact that I have promised to feed *either* your cat or your dog. I infer that satisficing validity is not a useful kind of validity. Definitions 1a and 1b, by contrast, do have (D6) as a consequence:[32] I do have an obligation to feed your cat if I have promised to feed *both* your cat and your dog.

I conclude that my definitions of strong and weak validity are preferable to all alternative definitions of pure imperative validity that I considered above. It may still seem mysterious, however, how one can use my definitions of strong and weak validity to decide whether any specific pure imperative argument is (strongly or weakly) valid. The mystery is lifted in the next section.

## 4. An equivalence theorem

### *4.1 The theorem and its significance*

Here is the fundamental result of this paper:

> Equivalence Theorem: Let $S$, $V$, and $C$ be respectively the satisfaction proposition, the violation proposition, and the context of the conjunction of the premisses of a pure imperative argument, and define similarly $S'$, $V'$, and $C'$ for the conclusion of the argument.
>
> (1) The argument is strongly valid exactly if: $V$ is necessary, or $S'$ entails $S$ and $V'$ entails $V$
> (2) The argument is weakly valid exactly if: $C'$ entails $C$ and $V'$ entails $V$

[32] This is strictly speaking not accurate, as I plan to explain in another paper: with certain modifications, Definition 1b has as consequences those variants of (D6) that correspond to what I call *weak* obligatoriness, and Definition 1a has as (additional) consequences those variants of (D6) that correspond to what I call *strong* obligatoriness (and similarly for (D1)–(D5) and weak versus strong bindingness). Weak obligatoriness corresponds to what may be called *means/ends* support: if it is a fact that I have promised to help you tomorrow, then the prescription you express by 'do not kill yourself today' is obligatory and is supported by the above fact, but is not supported weakly because the proposition that I help you tomorrow does not *entail* that I do not kill myself today (resurrection is — logically and metaphysically — possible). One might suggest subsuming means/ends support under a broadened notion of weak support by broadening the notion of entailment between propositions used in Definition 4. I reject this suggestion because means/ends support, even if common in practice, is not directly relevant to imperative logic: the argument from 'help me tomorrow' to 'do not kill yourself today' is not valid (because, to repeat, resurrection is possible).

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It is a consequence of this theorem that the validity of a pure imperative argument, which I defined in terms of the intuitively appealing but initially nebulous relation of support between reasons and prescriptions, can be captured in terms of the clear and precise relation of entailment between propositions. The theorem enables one to decide whether any specific pure imperative argument is (strongly or weakly) valid, provided that one can check whether the conditions specified in the theorem hold.[33] For example, the argument from 'if you love me, neither smoke nor drink' to 'if you love me, do not smoke' is weakly valid because the context $C'$ of the conclusion, namely the proposition that you love me, is the same as (and thus entails) the context $C$ of the premiss, and the violation proposition $V'$ of the conclusion, namely the proposition that you love me and you smoke, entails the violation proposition $V$ of the premiss, namely the proposition that you love me and either you smoke or you drink (or both). One can similarly check that the argument is not strongly valid. Note that to perform such checks one does not need to assume a substantive theory of reasons; this is as it should be, because a substantive theory of reasons lies beyond the scope of logic.[34]

Because my proof of the Equivalence Theorem is rather long, I give it in Appendix A. Next I examine some corollaries of the theorem.

### *4.2 Corollaries and further results*

As a first corollary of the Equivalence Theorem, I can now prove a result I announced in section 3.4, namely that every strongly valid pure imperative argument is also weakly valid. It suffices to show that, if $V$ is necessary, or $S'$ entails $S$ and $V'$ entails $V$, then $C'$ entails $C$ and $V'$ entails $V$. Indeed: (1) if $V$ is necessary, then so is $C$, and then $C'$ entails

[33] Given that the conditions specified in the theorem 'include' the condition that $V'$ entails $V$, which corresponds to obedience-validity (see Sect. B.1.2), I take those conditions to correspond to *variants* of obedience-validity; this is why I said in Sect. 1 that for each of the two kinds of pure imperative validity that I distinguish (namely strong and weak) there is an equivalent variant of obedience-validity.

[34] One might note that in Sect. 3 I made substantive claims about reasons; e.g. I claimed that the fact that I have promised to help you favours every proposition which entails that I help you over every different proposition which entails that I do not help you. But although those who reject my substantive claims about reasons may disagree with me on which pure imperative arguments are *sound* (in the sense of being valid *and* having binding premisses), they should still agree with me on which pure imperative arguments are (strongly or weakly) *valid*; in this sense, my approach to pure imperative inference does not depend on a substantive theory of reasons (except that my approach depends on Assumption 1 of Appendix A, a substantive assumption).

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$C$ and $V'$ entails $V$, and (2) if $S'$ entails $S$ and $V'$ entails $V$, then both $S'$ and $V'$ entail $C$ (since both $S$ and $V$ entail $C$), and then the disjunction of $S'$ with $V'$, namely $C'$, also entails $C$ (and $V'$ entails $V$). As a second corollary of the Equivalence Theorem, note that if a prescription is unobeyable (i.e. its violation proposition is necessary), then it strongly — and thus weakly — entails *every* prescription; this is analogous to the result in standard declarative logic that an impossible proposition entails every proposition.

Although I argued in section 3.4 that satisficing validity is not a useful kind of validity, it may be interesting to note that a proof very similar to my proof of the Equivalence Theorem shows that, if $V$ is not necessary, then satisficing validity is equivalent to the condition that $C'$ entails $C$ and $V$ entails $V'$.[35] This condition can be shown to entail that $S'$ entails $S$; similarly, in the previous paragraph the condition that $S'$ entails $S$ and $V'$ entails $V$ was shown to entail that $C'$ entails $C$. It is useful to mention these entailments explicitly in the following summary of equivalence results:

> Summary of Equivalence Results: Consider a pure imperative argument whose premisses have the ordered pair $<S, V>$ as their conjunction and whose conclusion is $<S', V'>$. If $V$ is necessary, then the argument is strongly and weakly valid. If $V$ is not necessary, then the argument is:
>
> (1) *strongly* valid exactly if: $V'$ entails $V$ and $S'$ entails $S$ (and thus $C'$ entails $C$)
>
> (2) *weakly* valid exactly if: $V'$ entails $V$ and $C'$ entails $C$
>
> (3) *satisficingly* valid exactly if: $V$ entails $V'$ and $C'$ entails $C$ (and thus $S'$ entails $S$)

If $<S, V>$ and $<S', V'>$ are *unconditional* prescriptions (more generally, if $C = C'$), then strong validity is trivial (because it amounts to $<S, V> = <S', V'>$) and weak validity amounts to satisfaction-validity (because it amounts to the condition that $S$ entails $S'$; equivalently, $V'$ entails $V$). Since for unconditional prescriptions satisfaction-validity is isomorphic to pure declarative validity, those who have claimed that pure imperative validity is isomorphic to pure declarative validity (e.g. Hofstadter and McKinsey 1939; Castañeda 1974, p. 85, 1975, p. 119) were partly right. But they were also mostly

[35] More specifically, it can be proven that (1) if $C'$ entails $C$ and $V$ entails $V'$, then the Argument (see Appendix A) is satisficingly valid, and that (2) if the Argument is satisficingly valid and $V$ is not necessary, then $C'$ entails $C$ and $V$ entails $V'$. To prove (1), replace $I''$ with $<S^* \vee S', C' \& \neg(S^* \vee S')>$ in the third part of the proof in Appendix A. To prove (2), use reasoning similar to that in the fourth part of the proof in Appendix A to show, for *reductio*, that $R$ in $W$ favours $V \& \neg V^*$ over $S \& V^*$ but also favours $S \& V^*$ over $V \& \neg V^*$.

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wrong, because they neglected the more interesting cases of pure imperative arguments, namely those in which the conclusion and the conjunction of the premisses are not both unconditional (more generally, do not have the same context).[36]

To provide further evidence for the usefulness of weak validity, I will now argue that weak validity is equivalent to what may be called *redundancy*-validity. Say that a pure imperative argument is *redundancy-valid* exactly if the conjunction of its conclusion with the conjunction of its premisses is the same as the conjunction of its premisses; informally, the conclusion is redundant in the sense that adding (i.e. conjoining) it to the conjunction of the premisses leaves that conjunction unchanged. To prove that weak validity and redundancy-validity are equivalent, I will need to be more specific than I have been so far in this paper about how the conjunction of prescriptions is to be understood. In a previous paper (Vranas 2008, pp. 538–41) I have defended the following definition:

> DEFINITION 6: The *conjunction* of given prescriptions (the *conjuncts*) is the prescription whose context is the disjunction of the contexts of the conjuncts and whose violation proposition is the disjunction of the violation propositions of the conjuncts

To prove that *a pure imperative argument is weakly valid exactly if it is redundancy-valid*, consider a pure imperative argument whose premisses have $<S, V>$ as their conjunction and whose conclusion is $<S', V'>$. The argument is redundancy-valid exactly if $<S, V> \,\&\, <S', V'> = <S, V>$; that is, exactly if $C \vee C' = C$ and $V \vee V' = V$; that is, exactly if $C'$ entails $C$ and $V'$ entails $V$ (see n. 18); that is, exactly if the argument is weakly valid. Note that a similar result holds for pure declarative arguments: a pure declarative argument is valid exactly if the conjunction of its conclusion with the conjunction of its premisses is necessarily equivalent to the conjunction of its premisses.

Having defined the conjunction of prescriptions, I can now explain why my definition of strong validity in terms of reasons supporting

[36] The isomorphism, for unconditional prescriptions, of weak validity to pure declarative validity suggests the following analogue of Definition 1 as a definition of pure declarative validity: a pure declarative argument is valid exactly if, necessarily, every reason that weakly supports the unconditional prescription whose satisfaction proposition is the conjunction of the premisses of the argument also weakly supports the unconditional prescription whose satisfaction proposition is the conclusion of the argument.

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the *conjunction* of the premisses of a pure imperative argument is preferable to a definition in terms of reasons supporting *every* premiss:

> DEFINITION 7: A pure imperative argument is *non-conjunctively strongly valid* exactly if, necessarily, every reason that strongly supports *every* premiss of the argument also strongly supports the conclusion of the argument

Non-conjunctive strong validity differs from strong validity partly because a reason can strongly support the conjunction of certain prescriptions without strongly supporting every conjunct. For example, the fact that I have promised to both run and smile strongly supports the conjunction of the prescriptions expressed by 'run' and by 'smile' (because, by Definition 6, this conjunction is just 'run and smile') but does not strongly support both conjuncts (because, if it strongly supports 'run', then by the dominance condition (1) it favours the proposition that I run and I do not smile over the proposition that I smile and I do not run, and if it strongly supports 'smile', then by the dominance condition (2) it favours the proposition that I smile and I do not run over the proposition that I run and I do not smile; but (1)&(2) contradicts the asymmetry of comparative favouring). It can now be seen that Definition 7, in contrast to my definition of strong validity, does *not* have the following as a consequence:

> (D7) Necessarily, a multiple-premiss pure imperative argument *A* is valid exactly if the (corresponding) single-premiss pure imperative argument *A′* is valid whose single premiss is the conjunction of the premisses of *A* and whose conclusion is the conclusion of *A*

I submit that any useful definition of pure imperative validity will have (D7) as a consequence (cf. Åqvist 1965, p. 184): one should be able to combine multiple premisses by conjunction into a single premiss and to split a single premiss which is a conjunction of certain conjuncts into multiple premisses (the conjuncts) without affecting validity, just as one can do for pure declarative arguments. To see why Definition 7 does not have (D7) as a consequence, note that the argument from 'run' and 'smile' to 'run' is trivially non-conjunctively strongly valid (necessarily, every reason that strongly supports both 'run' and 'smile' also strongly supports 'run') but the argument from 'run and smile' to 'run' is not non-conjunctively strongly valid (because, by the Equivalence Theorem, the latter argument is not strongly valid, and for *single*-premiss arguments non-conjunctive

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strong validity coincides with strong validity). I conclude that Definition 7 is not a useful definition of pure imperative validity.[37] (In a note I make some remarks on a variant of Definition 7 that corresponds to *weak* support.[38])

## 5. The validity of specific pure imperative arguments

Consider two classifications of pure imperative arguments: according to whether they are strongly or weakly valid, and according to whether they are intuitively valid. The first classification yields three groups of arguments, consisting of those arguments that are (i) both strongly and weakly valid, (ii) neither weakly nor strongly valid, and (iii) weakly but not strongly valid. (Recall that no argument is strongly but not weakly valid.) The second classification also yields three groups of arguments, consisting of those arguments that are (i) intuitively valid, (ii) intuitively invalid, and (iii) neither intuitively valid nor intuitively invalid. (I take it that no argument is both intuitively valid and intuitively invalid; if intuitions about the validity of an argument conflict and the conflict cannot be resolved, I take the argument to be neither intuitively valid nor intuitively invalid.) In this section I argue that these two classifications roughly coincide. More

[37] Definition 7 also fails to have as a consequence the other 'half' of (D7): although the argument from $I\&I'$ to $I\&I'$ is trivially non-conjunctively strongly valid, the argument from $I$ and $I'$ to $I\&I'$ need not be, because a reason can strongly support two prescriptions without strongly supporting their conjunction. For example, a fact that strongly supports the prescriptions ($I$) 'if you run, smile' and ($I'$) 'if you do not run, smile' need not strongly support $I\&I'$, which by Definition 6 is just 'smile': the satisfaction indifference condition need not hold, because the fact in question might favour the proposition that I smile and I run over the proposition that I smile and I do not run.

[38] Every reason that weakly supports the conjunction of two prescriptions also weakly supports both conjuncts. This is because, by Definition 6, the context of $I\&I'$ (where $I=<S, V>$ and $I'=<S', V'>$) is $C\vee C'$ and the violation proposition is $V\vee V'$; since both $C$ and $C'$ entail $C\vee C'$ and both $V$ and $V'$ entail $V\vee V'$, by the Equivalence Theorem $I\&I'$ weakly entails both $I$ and $I'$, so by Definition 1b every reason that weakly supports $I\&I'$ also weakly supports both $I$ and $I'$. It is not clear whether the converse holds, namely whether every reason that weakly supports two prescriptions also weakly supports their conjunction. (I can prove that it holds if at least one of the two prescriptions is unconditional, and more generally if the context of one of the two prescriptions entails the context of the other.) If the converse holds, then non-conjunctive *weak* validity — defined by uniformly replacing 'strongly' with 'weakly' in my formulation of Definition 7 — is equivalent to weak validity. If the converse does not hold, then there are prescriptions $I$ and $I'$ such that some reason weakly supports $I$ and $I'$ but does not weakly support $I\&I'$. But then the argument from $I$ and $I'$ to $I\&I'$ is not non-conjunctively weakly valid although the argument from $I\&I'$ to $I\&I'$ trivially is, so non-conjunctive weak validity does not have (D7) as a consequence and is thus not a useful kind of validity.

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specifically, I defend three theses. (1) Almost every pure imperative argument that is both strongly and weakly valid is intuitively valid (Sect. 5.1). (2) Every pure imperative argument that is neither weakly nor strongly valid is intuitively invalid (Sect. 5.2). (3) Almost every pure imperative argument that is weakly but not strongly valid is neither intuitively valid nor intuitively invalid (Sect. 5.3). If these three theses are correct, it is reasonable to infer that (a) clear intuitions about pure imperative validity are largely explained by a convergence between strong and weak validity, and that (b) conflicting intuitions about pure imperative validity are largely explained by a divergence (and a vacillation) between strong and weak validity.

Whenever in what follows I talk about a pure imperative argument, I let $S$, $V$, and $C$ be respectively the satisfaction proposition, the violation proposition, and the context of the conjunction of the premisses of the argument, and I define similarly $S'$, $V'$, and $C'$ for the conclusion of the argument. I call a pure imperative argument *trivial* exactly if $<S, V> = <S', V'>$, and *non-trivial* otherwise. I call a pure imperative argument *obeyable* exactly if the conjunction of its premisses is obeyable (i.e. its obedience proposition is possible), and *unobeyable* otherwise.

*5.1 Arguments that are both strongly and weakly valid*

In this subsection I defend the thesis that every obeyable pure imperative argument that is strongly (equivalently: both strongly and weakly) valid is intuitively valid. This thesis follows from two premisses. (1) Every obeyable pure imperative argument that is strongly valid is an instance of what may be called *strengthening the antecedent*; that is, the corresponding single-premiss argument can be expressed by 'if $A$ is true, let $B$ be true; so if $A\&A^*$ is true, let $B$ be true' (for some propositions $A$, $A^*$, and $B$). (2) Every pure imperative argument that is an instance of strengthening the antecedent is intuitively valid. I defend these two premisses in turn.

The first premiss holds because, given any obeyable pure imperative argument that is strongly valid (so that $S'$ entails $S$, $V'$ entails $V$, and $C'$ entails $C$), the corresponding single-premiss argument, namely the argument from 'if $C$ is true, let $S$ be true' to 'if $C'$ is true, let $S'$ be true', can be expressed by 'if $C$ is true, let $S$ be true; so if $C\&C'$ is true, let $S$ be true' (and is thus an instance of strengthening the antecedent). Indeed, the context of 'if $C\&C'$ is true, let $S$ be true', namely $C\&C'$, is $C'$ (since $C'$ entails $C$), and the satisfaction proposition, namely $C\&C'\&S$, is $C'\&S$, namely $(S'\&S) \vee (V'\&S)$, which is $S'$ (since $S'$

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entails $S$, and $V'$ entails $V$ and thus entails $\neg S$). It can be shown that, conversely, every pure imperative argument that is an instance of strengthening the antecedent is strongly valid. So an obeyable pure imperative argument *is strongly valid exactly if it is an instance of strengthening the antecedent.*

The second premiss holds because, given any pure imperative argument that is an instance of strengthening the antecedent, the corresponding single-premiss argument — which can be expressed by 'if $A$ is true, let $B$ be true; so if $A\&A^*$ is true, let $B$ be true' — is an argument from the conjunction of two prescriptions with incompatible contexts to one of the two conjuncts (and is thus intuitively valid). This is because the prescription expressed by 'if $A$ is true, let $B$ be true' is the conjunction of the prescriptions expressed by 'if $A\&A^*$ is true, let $B$ be true' and 'if $A\&\neg A^*$ is true, let $B$ be true'. (Indeed, by Definition 6, the context of the conjunction is $(A\&A^*) \vee (A\&\neg A^*)$, namely $A$, and the violation proposition of the conjunction is $(A\&A^*\&\neg B) \vee (A\&\neg A^*\&\neg B)$, namely $A\&\neg B$.) For example, the argument from 'if it rains, close the window' to 'if it rains and thunders, close the window' is intuitively valid because 'if it rains, close the window' is the conjunction of 'if it rains and thunders, close the window' and 'if it rains and does not thunder, close the window'. I am talking about *tutored* intuitions; so my claim that the above argument is intuitively valid is compatible with the claim (see Sect. 1) that some people — who were unfamiliar with my reasoning — have taken the argument to be invalid.[39]

Objecting to the second premiss, one might claim that the argument from 'do not wake me up' to 'if the house is on fire, do not wake me up' is an instance of strengthening the antecedent but is intuitively invalid (cf. Zellner 1971, p. 57; Sosa 1970, p. 217). In reply I distinguish two prescriptions that the sentence 'do not wake me up' can express, and two corresponding arguments that the objection can be about. First, the sentence 'do not wake me up' can express the unconditional

[39] On the (intuitive) validity of the above argument and of similar arguments see: Chellas 1969, pp. 100–1; Espersen 1967, pp. 77, 94; Green 1998, p. 720; Rescher 1966a, pp. 62–71, 90–1; Ross 1968, p. 176; Sosa 1964, p. 78, 1966b, pp. 233–4, 1967, pp. 62–3, 1970, pp. 216–18; Zellner 1971, p. 57. The fact that I am talking about *tutored* intuitions also provides a response to the claim that some people may take an argument which is an instance of strengthening the antecedent and has multiple premisses to be invalid because they do not see what the conjunction of the premisses is and thus they do not see that the argument is an instance of strengthening the antecedent.

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prescription whose violation proposition is the proposition that you wake me up; this prescription is violated if the house is on fire and you wake me up, and can also be expressed by 'do not wake me up, no matter what (e.g. regardless of whether the house is on fire)'. The argument that corresponds to this first prescription is indeed an instance of strengthening the antecedent, but is also intuitively valid because this first prescription is the conjunction of the prescriptions standardly expressed by 'if the house is on fire, do not wake me up' and 'if the house is not on fire, do not wake me up'. Second, the sentence 'do not wake me up' can express the prescription standardly expressed by 'if there is no emergency, do not wake me up'. The argument that corresponds to this second prescription is indeed intuitively invalid, but is not an instance of strengthening the antecedent because it is not strongly valid (given that the violation proposition of its conclusion, namely the proposition that the house is on fire and you wake me up, does not entail the violation proposition of its premiss, namely the proposition that there is no emergency and you wake me up). I conclude that in neither case do we have an argument that is both intuitively invalid and an instance of strengthening the antecedent, and the objection fails.[40]

Objecting further to the second premiss, one might claim that the argument from 'do not wake me up (no matter what)' to 'if you have conclusive reason to wake me up, do not wake me up' is clearly an instance of strengthening the antecedent but is intuitively invalid because its premiss can be binding but its conclusion cannot be. I reply that, contrary to appearances, the conclusion *can* be binding. Suppose, for example, that I have promised not to wake you up (no matter what). *As far as my promise is concerned*, what matters is whether I wake you up or not; it does not matter whether I have

[40] Objecting further to the second premiss, one might claim that the argument from 'marry him' to 'if you do not marry him, marry him' is an instance of strengthening the antecedent but is intuitively invalid: its conclusion is *unsatisfiable* (i.e. its satisfaction proposition is impossible), so — by a variant of the ought-implies-can principle — it cannot be binding even if the premiss is binding. (Similarly if the conclusion is replaced with the satisfiable prescription expressed by 'if you kill him, marry him'.) I reply that, although I accept the principle that no *unobeyable* prescription can be binding (see n. 67), I reject the principle that no *unsatisfiable* prescription can be binding: in n. 22 I argued (and Assumption 1 of Appendix A entails) that unsatisfiable prescriptions can be binding. This does not conflict with an acceptable variant of the ought-implies-can principle: you can marry him (or not kill him), so you *can* refrain from violating the prescription expressed by 'if you do not marry him, marry him' (or by 'if you kill him, marry him').

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conclusive reason to wake you up or not. So *relative* to the fact that I have promised not to wake you up, it is better if I have conclusive reason to wake you up and I do not wake you up than if I have conclusive reason to wake you up and I wake you up (although of course it need not be better *simpliciter*),[41] and this further objection to the second premiss also fails.[42]

This completes my defence of the thesis that every *obeyable* pure imperative argument that is strongly valid is intuitively valid. But what about *unobeyable* pure imperative arguments, like the one from 'prove that 2+2 is 5' to 'open the window'? All such arguments are strongly valid, and arguably they are also intuitively valid: 'prove that 2+2 is 5' can be expressed by 'open the window and do not open the window' (see n. 18). But even if one disagrees and claims that many such arguments are intuitively invalid, this is no more a problem for my

[41] In response one might argue that my reply does not work for the prescription expressed by 'if you have conclusive reason to wake me up *and there is no reason not to wake me up*, do not wake me up': if there is no reason not to wake you up, then the fact that I have promised not to wake you up is not such a reason (e.g. because I have not *freely* promised). In reply I can grant that the fact that I have promised not to wake you up *would not* have supported the above prescription *if* it had not been a reason not to wake you up, but it does not follow that the above fact, which by assumption *is* a reason not to wake you up, does not *actually* support the above prescription. In any case, here is a reason why the above prescription *can* be binding. Suppose it is a fact that the expected long-term consequences associated with the proposition that I have conclusive reason to wake you up, there is no reason not to wake you up, and I do not wake you up are better than the expected long-term consequences associated with the proposition that I have conclusive reason to wake you up, there is no reason not to wake you up, and I wake you up. This fact, I submit, supports the above prescription (but need not be a reason not to wake you up, so the above supposition is compatible with the proposition that no such reason exists).

[42] Here is yet another objection to the second premiss. Suppose it is a fact that one's chance of getting a job (1) is higher given that one studies engineering than given that one does not, but (2) is lower given that one studies both law and engineering than given that one studies law but not engineering. This fact supports the prescription expressed by 'study engineering' but does not support the prescription expressed by 'if you study law, study engineering'. So it is possible for a reason to support the premiss but not the conclusion of the argument from 'study engineering' to 'if you study law, study engineering', and the argument is intuitively invalid although it is an instance of strengthening the antecedent—or so the objection goes. In reply I propose a *reductio* of the objection: if the reasoning of the objection is accepted, then the absurd conclusion follows that for about *any* prescriptions $<S, V>$ and $<S', V'>$ the argument from $<S, V>$ to $<S', V'>$ is intuitively invalid. Indeed, it suffices to suppose, for some proposition $G$ to the effect that something good happens, that it is a fact that $P(G|S) > P(G|V)$ and $P(G|S') < P(G|V')$. (Even if $S$ entails $S'$ or vice versa (but not both), the conditional probabilities $P(G|S)$ and $P(G|S')$ need not be related.) So the objection fails; but why does it fail? Because it appeals neither to strong nor to weak support, but rather to what in n. 32 I called *means/ends* support; as I said in that note, means/ends support is not directly relevant to imperative logic.

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definitions of pure imperative validity than the claim that arguments like the one from 'you will prove that 2+2 is 5' to 'you will open the window' are intuitively invalid is a problem for the standard definition of pure declarative validity.

*5.2 Arguments that are neither weakly nor strongly valid*

In this subsection I defend the thesis that every pure imperative argument that is not weakly (equivalently: neither weakly nor strongly) valid is intuitively invalid. To defend this thesis, I partition the group of arguments that are not weakly valid — which, by the Equivalence Theorem, is the group of arguments for which $C'$ does not entail $C$ or $V'$ does not entail $V$ — into four subgroups, consisting of those arguments for which (i) $C'$ entails $C$ and $V'$ does not entail $V$, (ii) $C'$ does not entail $C$ and $C$ entails $C'$ (i.e. $C'$ *extends* $C$), (iii) $C'$ does not entail $C$, $C$ does not entail $C'$, and $C\&C'$ is possible (i.e. $C$ and $C'$ *overlap*), and (iv) $C'$ does not entail $C$, $C$ does not entail $C'$, and $C\&C'$ is impossible (i.e. $C$ and $C'$ are *incompatible*[43]). I examine these four subgroups in reverse order.

*5.2.1 First subgroup: C and C′ are incompatible*

I submit that every pure imperative argument that can be expressed by 'if $A$ is true, let $B$ be true; so if $A$ is false and $A^*$ is true, let $B$ be true' (for some propositions $A$, $A^*$, and $B$) is intuitively invalid. For example, the argument from 'if you love him, marry him' to 'if you do not love him and he proposes, marry him' is intuitively invalid. It can be shown that for *every* argument in the first subgroup the corresponding single-premiss argument can be expressed as above (by letting $A = C$, $A^* = C'$, and $B = S \vee S'$). I infer that every argument in the first subgroup is intuitively invalid. One might object by claiming that the argument from 'if you love him, marry him' to 'if you do not love him but you have conclusive reason to marry him, marry him' is intuitively valid. (Compare an analogous argument I examined in Sect. 5.1.) I reply that those who take this argument to be valid presumably do so not because they see any connection between its premiss and its conclusion, but rather because they take the conclusion to be necessarily supported by *every* reason and thus to follow

[43] The claim that $C'$ does not entail $C$ and $C$ does not entail $C'$ is entailed by the claim that $C\&C'$ is impossible (and that $C$ and $C'$ are possible, but recall that I am considering only non-empty prescriptions), so the former claim is redundant in the definition of the current subgroup.

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from *every* prescription.[44] Contrary to appearances, however, the conclusion is not necessarily supported by every reason; for example, it is not supported by the fact — suppose it is a fact — that you have promised not to marry him. Indeed, as far as your promise is concerned, what matters is whether you marry him or not; it does not matter whether you have conclusive reason to marry him or not. So *relative* to the fact that you have promised not to marry him, it is *not* better if (1) you do not love him, you have conclusive reason to marry him, and you marry him than if (2) you do not love him, you have conclusive reason to marry him, and you do not marry him; in other words, the above fact does not support the conclusion of the above argument. I conclude that the objection fails.

*5.2.2 Second subgroup: C and C′ overlap*

The propositions that you love him and that he loves you overlap (i.e. neither of them entails the other, and their conjunction is possible), and the argument from 'if you love him, marry him' to 'if he loves you, marry him' is intuitively invalid. Many similar examples can be adduced to support the generalization that every argument in the second subgroup is intuitively invalid. One might object by considering the following argument:

ARGUMENT A:
If you study engineering, do not study philosophy
So: If you study philosophy, do not study engineering

One might claim that Argument A is in the second subgroup (because the propositions that you study engineering and that you study philosophy overlap) but is intuitively valid. (Argument A is an instance of what may be called *contraposition*.[45]) In reply I will argue that (contrary to appearances) Argument A is *not* intuitively valid.[46]

[44] Alternatively, they might take the conclusion to be necessarily binding (i.e. they might take it to be necessary that some reason supports the conclusion). I reply that no prescription is necessarily binding because it is (logically) possible that no reasons exist.

[45] I call a pure imperative argument an instance of *contraposition* exactly if $<S', V'>$ is *a contrapositive* of $<S, V>$, namely exactly if $S' \& S$ is impossible and $V' = V$. In support of my definition of a contrapositive, note that it is natural to call the prescription expressed by 'if $S'$ or $V$ is true, let $S$ or $V$ be false' a contrapositive of the prescription expressed by 'if $S$ or $V$ is true, let $S'$ or $V$ be false', and that if $S' \& S$ is impossible then the former prescription is $<S', V>$ and the latter prescription is $<S, V>$ (indeed, the violation proposition of both prescriptions is $(S \vee V) \& (S' \vee V)$, which is $V$ if $S' \& S$ is impossible).

[46] I *need* to argue that Argument A is not intuitively valid because to reply to the objection it would not be enough to argue — as several people have argued (see: Beardsley 1944, p. 184;

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My argument has three premisses. (1) If Argument A is intuitively valid, then the 'converse' argument from 'if you study philosophy, do not study engineering' to 'if you study engineering, do not study philosophy' is also intuitively valid. (2) If the argument from $I$ to $I'$ and the converse argument from $I'$ to $I$ are both intuitively valid, then the contexts of $I$ and $I'$ are intuitively necessarily equivalent (because the satisfaction propositions of $I$ and $I'$ are intuitively necessarily equivalent, and so are the violation propositions). (3) The contexts of the prescriptions expressed by 'if you study engineering, do not study philosophy' and by 'if you study philosophy, do not study engineering' are *not* intuitively necessarily equivalent (because they overlap, otherwise Argument A is not in the second subgroup and the objection does not even get off the ground). From these three premisses it follows that Argument A is not intuitively valid. I conclude that the objection fails.

Here is an explanation of why one might — mistakenly — claim that Argument A is intuitively valid: one might take the premiss and the conclusion of Argument A to be the *same* prescription, because one might confuse the premiss of Argument A, namely the *conditional* prescription expressed by 'if you study engineering, do not study philosophy' (a prescription which is *avoided* if you do not study engineering), with the *unconditional* prescription expressed by 'let it be the case that if you study engineering you do not study philosophy' (a prescription which is *satisfied* if you do not study engineering, because its satisfaction proposition is the material conditional expressed by 'if you study engineering, you do not study philosophy'), and similarly for the conclusion of Argument A (cf. Vranas 2008, pp. 534–5). So one might confuse Argument A with the (intuitively valid) trivial argument from 'let it be case that if you study engineering you do not study philosophy' to 'let it be the case that if you study philosophy you do not study engineering'. In support of this explanation, note that one is less inclined to claim that an instance of contraposition is intuitively valid when one is less inclined to take its premiss and its conclusion to be the same prescription, as for example in the argument from 'if the volcano erupts, flee' to 'if you do not flee, let it not be the case that the volcano erupts'.

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Clarke 1985, p. 104; Fitch 1940; Makinson 1999, p. 37; cf. Clarke 1979a, pp. 609–10) — that *some* instances of contraposition are not intuitively valid: even a single intuitively valid instance of contraposition (with overlapping contexts) would refute my claim that every argument in the second subgroup is intuitively invalid.

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In response one might note that the argument from 'if the volcano erupts, flee' and 'smile or do not smile' to 'if you do not flee, let it not be the case that the volcano erupts' is an instance of strengthening the antecedent (because it turns out that the corresponding single-premiss argument can be expressed by 'let it not be case that the volcano erupts and you do not flee; so if you do not flee, let it not be the case that the volcano erupts and you do not flee').[47] So I must claim that this argument is intuitively valid; but then how can I also claim that the argument from 'if the volcano erupts, flee' to 'if you do not flee, let it not be the case that the volcano erupts' is intuitively invalid? How can the 'vacuous' prescription 'smile or do not smile' be responsible for the difference between an intuitively valid argument and an intuitively invalid one? I reply that — as I explain in note 47 — the prescription 'smile or do not smile' is responsible for the difference between an argument with an unconditional conjunction of premisses ('let it not be the case that the volcano erupts and you do not flee') and an argument with a conditional conjunction of premisses ('if the volcano erupts, flee'). This difference matters because the unconditional conjunction of premisses, unlike the conditional one, is the conjunction of the conclusion of both arguments ('if you do not flee, let it not be the case that the volcano erupts and you do not flee') with 'if you flee, let it not be the case that the volcano erupts and you do not flee', and thus intuitively entails the conclusion.

In response one might claim that the prescription 'smile or do not smile' is necessarily supported by every reason and is thus analogous to a necessary proposition: it is at least implicitly a premiss in every argument, so it should not matter to (intuitive) validity whether one

[47] The premiss 'smile or do not smile' can be replaced with 'if the volcano does not erupt, let it be the case that the volcano does not erupt', or more generally with any prescription whose violation proposition is impossible and whose context is entailed by the proposition that the volcano does not erupt. The fact that the conjunction of 'if the volcano erupts, flee' with 'smile or do not smile' is 'let it not be the case that the volcano erupts and you do not flee' follows from Definition 6 but can also be seen intuitively as follows (letting $R$, $F$, and $M$ be respectively the propositions that the volcano erupts, that you flee, and that you smile): 'if $R$ is true, let $F$ be true' & 'let $M \vee \neg M$ be true' = 'if $R$ is true, let $F$ be true' & ('if $R$ is true, let $M \vee \neg M$ be true' & 'if $R$ is false, let $M \vee \neg M$ be true') = ('if $R$ is true, let $F$ be true' & 'if $R$ is true, let $M \vee \neg M$ be true') & 'if $R$ is false, let $M \vee \neg M$ be true' = 'if $R$ is true, let $F\&(M \vee \neg M)$ be true' & 'if $R$ is false, let $M \vee \neg M$ be true' = 'if $R$ is true, let $F$ be true' & 'if $R$ is false, let $M \vee \neg M$ be true' = 'if $R$ is true, let $R\&\neg F$ be false' & 'if $R$ is false, let $R\&\neg F$ be false' = 'let $R\&\neg F$ be false'. (The prescriptions expressed by 'if $R$ is false, let $M \vee \neg M$ be true' and by 'if $R$ is false, let $R\&\neg F$ be false' are the same because their violation propositions, namely $\neg R\&\neg(M \vee \neg M)$ and $\neg R\&(R\&\neg F)$ respectively, are both impossible, and their contexts are the same, namely $\neg R$.)

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explicitly includes it in one's premisses or not. I reply that the analogy fails: 'smile or do not smile' is not necessarily supported by every reason. For example, it is not supported by the fact that it is better for your health if the volcano erupts and you flee than if the volcano erupts and you do not flee. (More generally, it is not supported by any reason which is *conditional* — see Appendix A — on the proposition that the volcano erupts or on any other contingent proposition.) I can grant that 'smile or do not smile' is binding and is thus analogous to a *true* (rather than a *necessary*) proposition, but this analogy would support my case: adding a true proposition to the premisses of an intuitively invalid pure declarative argument can yield an intuitively valid argument (e.g. add 'Paris is the capital of France' to the premiss of 'if Paris is the capital of France, then Paris is in France; so Paris is in France').

*5.2.3 Third subgroup: C′ extends C*

The proposition that you wake up extends the proposition that you wake up and you see a burglar (i.e. the latter entails the former but not vice versa), and the argument from 'if you wake up and you see a burglar, call the police' to 'if you wake up, call the police' is intuitively invalid. Many similar examples can be adduced to support the generalization that every argument in the third subgroup (i.e. every instance of what may be called *extending the context*) is intuitively invalid. One might object by considering the following argument:

ARGUMENT B:
If you study engineering, do not study philosophy
So: Do not study both engineering and philosophy

One might claim that Argument B is in the third subgroup (because the context of its conclusion is necessary and thus extends the proposition that you study engineering) but is intuitively valid. In reply I will argue that (contrary to appearances) Argument B is *not* intuitively valid. My argument has three premisses. (1) If Argument B is intuitively valid, then so is the argument from the prescription expressed by 'if you study engineering, do not study both engineering and philosophy' (this prescription turns out to be the premiss of Argument B) to the conjunction of the prescriptions expressed by 'if you study engineering, do not study both engineering and philosophy' and by 'if you do not study engineering, do not study both engineering and philosophy' (this conjunction turns out to be the conclusion of Argument B). (2) If the argument from $I$ to $I\&I^{*}$ is intuitively

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valid, then so is the argument from *I* to *I** (for any prescriptions *I* and *I**). (3) The argument from 'if you study engineering, do not study both engineering and philosophy' to 'if you do not study engineering, do not study both engineering and philosophy' is not intuitively valid. From these three premisses it follows that Argument B is not intuitively valid. I conclude that the objection fails.

In response one might contest the third premiss by claiming that the argument from 'if you study engineering, do not study both engineering and philosophy' to 'if you do not study engineering, do not study both engineering and philosophy' *is* intuitively valid because its conclusion — which can also be expressed by 'if you do not study engineering, do not study engineering' — is *unviolable* (i.e. its violation proposition is impossible) and thus intuitively follows from *every* prescription. In reply I reject the claim that an unviolable prescription intuitively follows from every prescription: the unviolable prescription expressed by 'if you murder, murder' does not intuitively follow from the prescription expressed by 'if you do not murder, do not murder' or from the prescription expressed by 'if he proposes, marry him'. I conclude that the response fails.[48]

The above considerations suggest an explanation of why one might — mistakenly — claim that Argument B is intuitively valid: since the conclusion of Argument B amounts to the conjunction of the premiss with a 'vacuous' (i.e. unviolable) prescription (namely 'if you do not study engineering, do not study engineering'), one might think that the conclusion of Argument B does not go 'beyond' the premiss. As I suggested in the previous paragraph, however, the vacuous conjunct of the conclusion of Argument B does go beyond the premiss. By analogy with what I said about Argument A (Sect. 5.2.2), here is also a second explanation of why one might claim that Argument B is intuitively valid: one might confuse Argument B with the (intuitively valid) trivial argument from 'let it be the case that if you study engineering you do not study philosophy' to '(let it be the case that you) do not study both engineering and philosophy'.

[48] By analogy with what I said about Argument A (Sect. 5.2.2), here is also a second argument for the conclusion that Argument B is not intuitively valid. (1) The converse argument is intuitively valid (because it is an instance of strengthening the antecedent: it can be expressed by 'do not study both engineering and philosophy; so if you study engineering, do not study both engineering and philosophy'). So (2) if Argument B is also intuitively valid, then the contexts of its premiss and its conclusion are intuitively necessarily equivalent; but (3) they are not.

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Finally, here is a third explanation of why one might claim that Argument B is intuitively valid: one might take Argument B to have an extra, implicit premiss, namely 'either study or do not study engineering'. So one might confuse Argument B with the (intuitively valid) trivial argument from 'if you study engineering, do not study philosophy' and 'either study or do not study engineering' to 'do not study both engineering and philosophy'. (The latter argument is trivial because, as one can show by using Definition 6, the conjunction of its premisses is the same as its conclusion.) This confusion is understandable: in a context in which I am only told (e.g. by an advisor) not to study philosophy if I study engineering, I am typically justified in believing that I have also been implicitly told to either study or not study engineering (as I see fit). So in a context in which the only explicit premiss is the premiss of Argument B, I am typically justified in considering 'either study or do not study engineering' as an extra, implicit premiss. But the fact remains that the conclusion of Argument B intuitively follows from the premiss of Argument B *together* with the extra premiss, not from the premiss of Argument B *alone*.[49]

*5.2.4 Fourth subgroup: $C'$ entails $C$ and $V'$ does not entail $V$*

If $V'$ does not entail $V$, then either (1) $V'$ extends $V$ (i.e. $V'$ does not entail $V$ and $V$ entails $V'$), or (2) $V$ and $V'$ overlap (i.e. $V'$ does not entail $V$, $V$ does not entail $V'$, and $V\&V'$ is possible), or (3) $V$ and $V'$ are possible but incompatible (i.e. $V'$ does not entail $V$, $V$ does not entail $V'$, and $V\&V'$ is impossible). Here are examples of arguments (with $C' = C$) that correspond to these three cases. (1) 'If you love him, marry him; so if you love him, marry him and kill him.' (2) 'If you love him, marry him; so if you love him, kill him.' (3) 'If you love him, marry him; so if you love him, do not marry him.' All these arguments are intuitively invalid, and many similar examples can be

[49] One might alternatively take the implicit premiss to be 'if you do not study engineering, either study or do not study philosophy' (which is the same as 'if you do not study engineering, do not study engineering'), or more generally any unviolable prescription whose context is entailed by the proposition that you do not study engineering (because the conjunction of any such prescription with the premiss of Argument B turns out to be the same as the conclusion of Argument B). Note also that a similar explanation works for Argument A: one might confuse Argument A with the argument from 'if you study engineering, do not study philosophy' and 'either study or do not study engineering' to 'if you study philosophy, do not study engineering'. The latter argument is intuitively valid because it turns out to be an instance of strengthening the antecedent (cf. n. 47 and corresponding text).

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adduced to support the generalization that every argument in the fourth subgroup is intuitively invalid. One might object by claiming that the argument from 'believe that not all ravens are black' to 'believe that some ravens are not black' is in the fourth subgroup — because it is possible for you to believe that ($P$) not all ravens are black without believing that ($P'$) some ravens are not black — but is intuitively valid because, necessarily, every reason for you to believe $P$ is a reason for you to believe $P'$. I reply that, *if* it is indeed possible for you to believe $P$ without believing $P'$ (e.g. because believing $P$ and believing $P'$ amount to having different mental representations), then it is also possible that some reason for you to believe $P$ is not a reason for you to believe $P'$; an example of such a reason is the fact that your daughter's life will be saved exactly if you believe $P$ without believing $P'$. In response one might claim that the above fact is a reason for you to *make* yourself believe $P$ but is not a reason for you to *believe* $P$ because no non-epistemic reasons for belief can exist. In reply note first that in the above example your daughter's life will be saved exactly if you *believe* $P$ without believing $P'$, even if you believe $P$ without *making* yourself believe it (e.g. because someone else makes you believe it). Moreover, I do not think it is really controversial that non-epistemic reasons for belief can exist (cf. e.g. BonJour 1985, pp. 6–7; Williamson 2000, p. 207); I understand those who maintain that pragmatic considerations can never justify a belief as denying that pragmatic considerations can be *epistemic* reasons for belief, not as denying that pragmatic considerations can be non-epistemic reasons for belief.[50]

[50] It is arguably also possible that some *epistemic* reason for you to believe $P$ is not a reason for you to believe $P'$; an example is the fact that an expert ornithologist confidently tells you 'not all ravens are black, but it is false that some ravens are not black' (assume that the expert misspoke, intending to say 'white' instead of 'not black', and that your logical skills are so poor that you fail to recognize the inconsistency). In response one might claim that the fact that an expert confidently asserts an impossible proposition is not a reason to believe that proposition. I reply that the fact that an expert mathematician confidently asserts a mathematical proposition that will be later on discovered to be false and thus impossible *is* now a reason to believe that proposition. In response one might claim that the case of the ornithologist is relevantly different: the ornithologist's assertion is *obviously* logically inconsistent, so you *ought* to recognize the inconsistency even if in fact you do not. I reply that what is obvious to one person need not be obvious to another, and whether one ought to recognize an inconsistency depends on one's epistemic situation and abilities: if your IQ is 50, it need not be true that you ought to recognize the inconsistency in the ornithologist's assertion. And if your IQ is not 50, it *could* have been 50; all I need is the *possibility* that some epistemic reason for you to believe $P$ is not a reason for you to believe $P'$.

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*5.3 Arguments that are weakly but not strongly valid*

In this subsection I defend the thesis that almost every pure imperative argument that is weakly but not strongly valid is neither intuitively valid nor intuitively invalid. This thesis follows from two premisses. (1) Every pure imperative argument that is weakly but not strongly valid is an instance of what may be called *strengthening the antecedent and strictly weakening the consequent* (or *strengthening/weakening* for short); that is, the corresponding single-premiss argument can be expressed by 'if $A$ is true, let $B$ be true; so if $A\&A^*$ is true, let $B \vee B^*$ be true' (for some propositions $A$, $A^*$, $B$, and $B^*$ such that $A\&A^*\&B^*\&\neg B$ is possible — if it is impossible, it turns out (see n. 51) that the argument is strongly valid). (2) Almost every instance of strengthening/weakening is neither intuitively valid nor intuitively invalid. I defend these two premisses in turn.

To defend the first premiss, note first that a pure imperative argument is weakly but not strongly valid exactly if $C'$ entails $C$, $V'$ entails $V$, $V$ is not necessary, and $S'$ does not entail $S$. (This can be shown by using the Equivalence Theorem.) Now given any such argument, the corresponding single-premiss argument, namely the argument from 'if $C$ is true, let $S$ be true; so if $C'$ is true, let $S'$ be true', can be expressed by 'if $C$ is true, let $S$ be true; so if $C\&C'$ is true, let $S \vee (S'\&\neg S)$ be true' (and is thus an — obeyable — instance of strengthening/weakening; note that $A\&A^*\&B^*\&\neg B$, namely $C\&C'\&(S'\&\neg S)\&\neg S$, is $S'\&\neg S$, which is possible because $S'$ does not entail $S$). Indeed, the context of 'if $C\&C'$ is true, let $S \vee (S'\&\neg S)$ be true', namely $C\&C'$, is $C'$ (since $C'$ entails $C$), and the satisfaction proposition, namely $C\&C'\&(S \vee (S'\&\neg S))$, is $C'\&(S \vee S')$, namely $(S'\&(S \vee S')) \vee (V'\&(S \vee S'))$, which is $S'$ (since $V'$ entails $V$ and thus entails $\neg S$). It can be shown that, conversely, every pure imperative argument that is an obeyable instance of strengthening/weakening is weakly but not strongly valid.[51] So a pure imperative argument *is weakly but not strongly valid exactly if it is an obeyable instance of strengthening/weakening.*

To defend the second premiss, I will first examine certain special instances of strengthening/weakening. Say that a pure imperative

[51] Indeed: if a pure imperative argument is an obeyable instance of strengthening/weakening, then $C'$ entails $C$ (because $C' = A\&A^*$ and $C = A$), $V'$ entails $V$ (because $V' = A\&A^*\&\neg B\&\neg B^*$ and $V = A\&\neg B$), $V$ is not necessary (because the argument is obeyable), and $S'$ does not entail $S$ (because $S' = A\&A^*\&(B \vee B^*)$ and $S = A\&B$, so $S'\&\neg S = A\&A^*\&(B \vee B^*)\&(\neg A \vee \neg B) = A\&A^*\&B^*\&\neg B$, which is possible), so the argument is weakly but not strongly valid.

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argument is an instance of *strictly weakening the consequent* exactly if the corresponding single-premiss argument can be expressed by 'if $A$ is true, let $B$ be true; so if $A$ is true, let $B \vee B^*$ be true' (for some propositions $A$, $B$, and $B^*$ such that $A \& B^* \& \neg B$ is possible). A widely discussed instance of strictly weakening the consequent is the following argument:

ARGUMENT R:
Post the letter
So: Post the letter or burn it

(The claim that the letter is burned is understood as entailing that the letter is not posted.) Many people — following Ross (1941, p. 62) — have taken Argument R and similar arguments to be intuitively invalid, but many other people have disagreed.[52] I will argue that Argument R and similar arguments are neither intuitively valid nor intuitively invalid.

A common reply to the claim that Argument R is intuitively invalid appeals to an analogy with declarative logic:

> It is true that any army officer who said to a subordinate "Post the letter; so post it or burn it!" would likely put his sanity under suspicion. But so would any housewife who solemnly announced "Dinner is served; hence dinner is served or the moon is yellow," unless she were thought to be joking. (Sosa 1966a, p. 212; cf. Castanẽda 1974, pp. 95–6)

The point is, I take it, that the typical inappropriateness of *publicly expressing* the above inferences — an inappropriateness which, according to Hare (1967, pp. 311–14), is due to the violation of a Gricean conversational maxim — casts no doubt on the intuitive validity of the arguments on which the inferences are based. One might respond that

[52] On the (intuitive) validity of Argument R and of similar arguments see: Adler 1980, p. 102; Aloni 2003, p. 57; Belnap, Perloff, and Xu 2001, pp. 83–5; Bennett 1970, p. 318; Bergström 1962, p. 40; Bohnert 1945, p. 313; Castañeda 1974, pp. 95–6; Chellas 1971, pp. 124, 127; Edgley 1969, pp. 169–76; Gombay 1965, p. 61, 1967, pp. 145–7; Green 1998, p. 718; Hall 1952, pp. 130, 137; Hansen 2008, pp. 25–31; Hare 1949, pp. 32–3, 1954, pp. 267–8, 1967, pp. 309–14; Hintikka 1977; Holdcroft 1978, pp. 111–16; Keene 1966, p. 60; Kenny 1966, p. 67; Mastop 2005, pp. 106–7; McArthur and Welker 1974, pp. 238–9; Moser 1956, p. 204; Niiniluoto 1986, pp. 115–16; Nolan 1977, p. 84; Ramírez 2003, pp. 20, 248–9; Rescher 1966a, pp. 115–17; Ross 1968, p. 161; Segerberg 1990, pp. 203–4, 220, 2005, p. 1; Sosa 1966a, p. 212; Storer 1946, p. 27, n. 2; Tammelo 1975, p. 41; Vanderveken 1990, p. 162; Wedeking 1969, pp. 61–2; Weinberger 1958, pp. 77–8; Williams 1963, p. 32; Zellner 1971, pp. 33–41. Argument R should be distinguished from its deontic variants (e.g. 'you are obligated to post the letter; so you are obligated to post the letter or burn it'); similarly, the imperative variant of 'Ross's paradox' — which arises from combining the claims that Argument R is intuitively invalid and that it comes out valid on certain plausible accounts of pure imperative validity — should be distinguished from deontic variants of the paradox.

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some inferences based on Argument R are problematic even if they are not publicly expressed: if I burn the letter because I infer, from the premiss via the conclusion of Argument R, that my burning the letter is permitted, am I not guilty of faulty reasoning? I am, but one might reply that the fault does not lie in inferring the conclusion from the premiss of Argument R; the fault lies instead in failing to notice that, according to the *premiss* of Argument R, my burning the letter is *not* permitted (cf. Hare 1954, pp. 267–8; Rödig 1972, pp. 184–5). Still, according to the *conclusion* of Argument R, my burning the letter *is* permitted, so a puzzle remains: how can anything be permitted according to the conclusion but not according to the premiss of an intuitively valid pure imperative argument?[53]

One might attempt to circumvent the puzzle by denying that my burning the letter is permitted according to the conclusion of Argument R. In order to deny this, one might distinguish two prescriptions that the imperative sentence 'post the letter or burn it' can express: the *choice-offering* prescription expressed by 'post the letter or burn it — at your choice', and the *alternative-presenting* prescription expressed by 'post the letter or burn it — not at your choice, but on the basis of further instructions'.[54] My burning the letter is permitted according to the choice-offering but not according to the alternative-presenting prescription, and one might claim that the conclusion of Argument R is the alternative-presenting prescription. I reply that the argument from 'post the letter' to the alternative-presenting prescription is not weakly valid (because if I choose to post the letter the conclusion of the argument is violated but the premiss is not) and is thus irrelevant to present concerns. So I take the conclusion of Argument R to be the choice-offering prescription, and the above attempt to circumvent the puzzle fails.

In response to the puzzle, one might argue that if something is permitted according to the conclusion but not according to the

[53] The puzzle is not dissolved by arguing that someone who expresses the premiss is not committed to expressing the conclusion and thus need not grant permission to do whatever is permitted according to the conclusion: the puzzle concerns what is permitted according to the conclusion, not whether anyone grants a relevant permission.

[54] The terms 'choice-offering' and 'alternative-presenting' were introduced by Rescher and Robison (1964, p. 179). On the application of the distinction to Ross's paradox see: Aloni 2003, p. 57; Rescher 1966a, pp. 115–17; Ross 1968, p. 161; Zellner 1971, pp. 33–41. It is not clear whether the sentence 'post the letter or burn it — not at your choice, but on the basis of further instructions' expresses a prescription at all, and if it does, what exactly the prescription amounts to (cf. Vranas 2008, pp. 542–3), but for what follows in the text I only need the claim that, if a prescription is expressed, it is violated if I choose to post the letter.

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premiss of a pure imperative argument, then it need not be permitted *simpliciter* (i.e. all-things-considered permitted), so there is no problem with accepting that the argument is intuitively valid: this acceptance does not license anything that the premiss forbids.[55] But then, one might ask, what is the point of inferences based on arguments like Argument R? If I must take into account that my burning the letter is not permitted according to 'post the letter', and thus I am not (all-things-considered) allowed to exercise the choice (between posting and burning the letter) that 'post the letter or burn it' gives me, then what is the point of inferring 'post the letter or burn it' from 'post the letter'? In reply one might similarly ask what is the point of inferring 'he posted the letter or burned it' from 'he posted the letter', and might claim that an imperative (like a declarative) argument can be intuitively valid even if only (or primarily) pointless inferences are based on it. This reply, however, might be considered unsatisfactory: those who are interested in a *useful* definition of pure imperative validity might deny that an argument on which only (or primarily) pointless inferences are based can be intuitively valid. They might claim instead that the conclusion of an intuitively valid pure imperative argument will stand on its own feet, in the sense that, necessarily, whatever is permitted according to the conclusion will also be permitted simpliciter (provided that something is permitted according to the conjunction of the premisses exactly if it is permitted simpliciter), and they might conclude that Argument R is not intuitively valid. We have thus reached a conflict of intuitions. I infer that Argument R and similar arguments are neither intuitively valid nor intuitively invalid.

[55] One might object that in some cases accepting that the argument is intuitively valid does license something that the premiss forbids. Suppose e.g. that you are given a three-question quiz whose single instruction is: 'answer exactly as many questions as there are non-prime numbers in the set {11110, 11111, 11112}'. Suppose also you can immediately see that 11110 and 11112 are not prime (since they are even), so you accept as intuitively valid the argument from the above instruction to 'answer at least two questions'. Suppose finally you have no way of finding out whether 11111 is prime. Then it is reasonable for you to decide to answer exactly two questions. In this example, your answering exactly two questions is not permitted according to the premiss (because it turns out that 11111 is not prime: it is 41 times 271), but is permitted according to the conclusion and is also permitted simpliciter if the argument is accepted as intuitively valid — or so the objection goes. One might reply that your answering exactly two questions is *subjectively* (and is not *objectively*) permitted simpliciter; but it is also subjectively (and is not objectively) permitted according to the premiss, so accepting that the argument is intuitively valid does not license anything that the premiss forbids (as long as licensing and forbidding are understood either both subjectively or both objectively).

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To sum up: the debate concerning the intuitive validity of Argument R (and of similar arguments) hinges on whether (1) something (e.g. burning the letter) can be permitted according to the conclusion but not according to the conjunction of the premisses of an intuitively valid pure imperative argument, and this in turn hinges on whether, (2) necessarily, whatever is permitted according to the conclusion of an intuitively valid pure imperative argument is also permitted simpliciter (provided that something is permitted according to the conjunction of the premisses of the argument exactly if it is permitted simpliciter). (Accepting (2) leads one to reject (1) and thus to conclude — via the premiss that burning the letter is permitted according to the conclusion but not according to the premiss of Argument R — that Argument R is not intuitively valid, but rejecting (2) commits one to accepting (1) and thus removes the only real obstacle I see to accepting that Argument R is intuitively valid.[56]) Now one might plausibly hold that, for an unconditional prescription, the claim that something is permitted according to the prescription exactly if it is permitted simpliciter amounts to the claim that the prescription is all-things-considered strongly binding. If so, then (2) follows from the claim that, (3) necessarily, the conclusion of an intuitively valid pure imperative argument is all-things-considered strongly binding if the conjunction of the premisses is, and this in turn follows from the claim that, (4) *necessarily, only strongly valid pure imperative arguments are intuitively valid*.[57] But then accepting that strong validity is necessary for intuitive validity commits one to accepting (2), whereas accepting that weak validity is sufficient for intuitive validity allows one to reject (2). In this sense, conflicting intuitions concerning Argument R (and similar arguments) can be explained by a vacillation between strong and weak validity.

[56] More precisely, it can be shown that the negation of (2) is equivalent to: (1*) possibly, something is permitted according to the conclusion but not according to the conjunction of the premisses of an intuitively valid pure imperative argument which is such that something is permitted according to the conjunction of its premisses exactly if it is permitted simpliciter. (1*) entails (1); moreover, accepting (1) leads one to accept (1*) because one may assume that the conjunction of the premisses of some argument satisfying (1) is all-things-considered strongly binding (see below in the text).

[57] Strictly speaking, (3) follows from the conjunction of (4) with the variant of (D2) (see Sect. 2) that corresponds to strong bindingness, and (3) and (4) are understood as restricted to arguments with unconditional conclusions and conjunctions of premisses (a restriction that does not matter for present purposes, since it is satisfied by Argument R and similar arguments).

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One might try to resolve the conflict of intuitions concerning Argument R by claiming that the argument is intuitively valid because it can be expressed by 'post the letter or burn it, and post the letter or do not burn it; so post the letter or burn it'. So Argument R is not only an instance of *imperative disjunction introduction* (namely an argument to the disjunction of two prescriptions from one of the disjuncts), but is also an instance of *imperative conjunction elimination* (namely an argument from the conjunction of two prescriptions to one of the conjuncts). In response one might claim that some instances of imperative conjunction elimination, for example the widely discussed argument from 'put on your parachute and jump out' to 'jump out', are not intuitively valid.[58] Strictly speaking, however, this argument is not an instance of imperative conjunction elimination if the satisfaction proposition of its premiss is the proposition that you will put on your parachute and *then* you will jump out: this proposition is not the conjunction of the propositions that you will put on your parachute and that you will jump out, but is rather the existentially quantified claim that, for some future times $t$ and $t'$ such that $t'$ is later than $t$, at $t$ you will put on your parachute and at $t'$ you will jump out. In response one might change the example: the argument from 'tutor both my daughter and my son' to 'tutor my son' is clearly an instance of imperative conjunction elimination, but one might claim that it is not intuitively valid because its conclusion permits you to tutor *only* my son. Of course one might reply that your tutoring only my son (like your jumping out without having put on your parachute) is not permitted according to the premiss and need not be permitted simpliciter; so we are back to the discussion of the previous paragraphs, and the conflict of intuitions reemerges. I conclude that the observation that Argument R is an instance of

[58] On the (intuitive) validity of the above argument and of (alleged) instances of imperative conjunction elimination see: Adler 1980, p. 68; Castañeda 1960b, p. 162, n. 22; Edgley 1969, pp. 170–4; Hall 1952, pp. 129, 130, n. 1, 136–7; Hamblin 1987, pp. 73–5; Hansen 2008, pp. 31–5; Hare 1949, p. 32, 1954, pp. 267–8; Holdcroft 1978, pp. 113–14; MacKay 1971, pp. 93–4; Menger 1939, p. 59; Ramírez 2003, pp. 252–4; Rescher and Robison 1964, p. 177; Ross 1941, p. 68, 1968, p. 163; Segerberg 1990, p. 217; Tammelo 1975, p. 39; Vanderveken 1990, p. 161; von Wright 1963, p. 181; Weinberger 1958, p. 24; Zellner 1971, pp. 31–2. Gombay (1965, p. 61) attempts to distinguish 'do x; so, do x or do y' from 'do x and do y; so, do x' by claiming that the former but not the latter argument corresponds to a set of commands that is 'sequentially inconsistent', in the sense that some 'obedience-possibility of one member is logically inconsistent with every obedience-possibility of another member': doing y but not x is logically inconsistent with doing x. I reply that a similar point holds about the latter argument: doing x but not y is logically inconsistent with doing both x and y. So Gombay's attempt fails.

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imperative conjunction elimination leaves unaffected my claim that Argument R and similar arguments are neither intuitively valid nor intuitively invalid.[59]

Besides Argument R, another widely discussed argument that is weakly but not strongly valid and that has been thought to be intuitively invalid is the argument — call it *Argument C* — from 'if you read the book, (then) come to see me' and 'read the book' to 'come to see me' (Castañeda 1958, p. 43). According to Castañeda, Argument C is invalid because a 'student will be drawing the wrong conclusion if [without reading the book] … he goes to see his teacher on the grounds that the latter issued the orders formulated in the premisses of [Argument C]' (1958, p. 44). My reply should come as no surprise: although going to see the teacher without (first) reading the book is permitted according to the conclusion of Argument C, it is not permitted according to the conjunction of the premisses (and it need not be permitted simpliciter). Indeed, by using Definition 6 one can show that the conjunction of the premisses of Argument C is 'read the book and (then) come to see me', so we are back to the discussion of the previous paragraph.[60] Similar remarks apply to the argument from 'use an axe or a saw' and 'do not use an axe' to 'use a saw' (cf. Hare 1949, p. 31, 1967, pp. 314–17; Bennett 1970; Bergström 1962, pp. 33–5, 40; Moutafakis 1975, pp. 74–5; Peters 1949, p. 540;

[59] My rejection of the claim that *every* instance of imperative conjunction elimination is intuitively valid is compatible with my acceptance (in Sect. 5.1) of the claim that *some* instances of imperative conjunction elimination are intuitively valid, namely those instances in which the two conjuncts of the premiss have incompatible contexts.

[60] On the (intuitive) validity of Argument C and of similar arguments — including (variants of) the argument from 'love your neighbour if you love yourself' and 'love yourself' to 'love your neighbour' — see: Bennett 1959, p. 265, 1970, pp. 315, 318; Bohnert 1945, p. 313; Castañeda 1960a, p. 28, 1960b, pp. 155, 169–70, 1963, p. 241, 1970, p. 444; Chaturvedi 1980, p. 477; Clarke 1970, pp. 101–2; Duncan-Jones 1952, p. 198; Frey 1957, p. 466; Gensler 1990, p. 197, 1996, p. 183; Green 1998, p. 720; Grelling 1939, p. 45; Grue-Sörensen 1939, p. 197; Hare 1969, p. 62; Hempel 1941, p. 106; Jørgensen 1938, p. 290; Kalinowski 1972, p. 77, n. 1; Ledent 1942, p. 270; MacKay 1969, pp. 148–9, 1971, pp. 92–3; Moser 1956, pp. 205–6; Rand 1939, p. 318, 1962, p. 248; Rescher 1966a, pp. 85, n. 11, 87; Ross 1941, p. 67, 1968, pp. 166–7; Vanderveken 1990, p. 60; Vetter 1971, p. 77; Weinberger 1958, pp. 45–6. Sosa (1970, p. 223) argues in effect that Argument C is invalid because, if it were valid, then from its premisses together with the premisses of the valid argument — call it *Argument C** — from 'if you do not read the book, do not come to see me' and 'you do not read the book' to 'do not come to see me' the self-contradictory conclusion 'come to see me and do not come to see me' would follow (cf. Chisholm 1963, pp. 34–5). In reply I deny that Argument C* is valid; but since this is a *mixed-premiss* imperative argument ('you do not read the book' is a *proposition*), I defend my view in a sequel to this paper (cf. n. 63).

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Williams 1963; Zellner 1971, pp. 36–41): the conjunction of its premisses turns out to be 'use a saw and do not use an axe'. These remarks can be generalized to every pure imperative argument that is an obeyable instance of strictly weakening the consequent, so I conclude that every such argument is neither intuitively valid nor intuitively invalid.[61]

The above remarks can also be generalized to pure imperative arguments that are instances of strengthening/weakening without being instances of strictly weakening the consequent. For example, my discussion of the argument from 'post the letter' to 'post the letter or burn it' applies, *mutatis mutandis*, to the argument from 'post the letter' to 'if it rains, post the letter or burn it'. One might argue, however, that certain other instances of strengthening/weakening are intuitively invalid. Consider, for example, the argument — call it *Argument D* — from 'if you are not a soldier, enlist' and 'if you enlist, wear a soldier's uniform' to 'if you are not a soldier, wear a soldier's uniform'. Argument D is an instance of strengthening/weakening because the conjunction of its premisses turns out to be 'if you are not a soldier or you enlist, enlist and wear a soldier's uniform'. But the question of whether this conjunction intuitively entails 'if you are not a soldier, wear a soldier's uniform' is analogous to the question of whether 'enlist and wear a soldier's uniform' intuitively entails 'wear a soldier's uniform': the two questions give rise to similar conflicts of intuitions. One might argue, however, that Argument D is intuitively invalid because its conclusion need not be (weakly) binding even if its premisses are: even if there is a reason for you to enlist given that you are not a soldier and there is a reason for you to wear a soldier's uniform given that you *in fact* enlist, if you *do not* in fact enlist then there need not be a reason for you to wear a soldier's uniform given that you are not a soldier (cf. Ross 1941, p. 67, n. 1; a

[61] Similar remarks apply to the argument from 'if you murder, repent' to 'if you murder, murder' (cf. Świrydowicz 1988, p. 235): its conclusion can also be expressed by 'if you murder, repent or do not repent'. Another (alleged) instance of strictly weakening the consequent that one might take to be intuitively invalid is the argument from 'apologize for having insulted Harry' to 'let it be the case that you have insulted Harry' (cf. Zellner 1971, p. 47). But *if* the proposition that you apologize for having insulted Harry entails that you have insulted Harry (as it must, if the argument is to be an instance of strictly weakening the consequent), then the premiss of the argument can be expressed by 'let it be the case that you have insulted Harry and that you apologize for having insulted Harry', so we are back to my discussion of imperative conjunction elimination. Alternatively, if the sentence 'apologize for having insulted Harry' expresses the prescription expressed by 'if you have insulted Harry, apologize for having insulted him', the corresponding argument is indeed intuitively invalid but is also not weakly valid (see Sect. 5.2.3).

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similar point can be made about Argument C).[62] I reply that, since Argument D is weakly valid, its conclusion must be weakly binding if the *conjunction* of its premisses is; I can grant that the conclusion need not be weakly binding if *every* premiss is, but I have already argued against defining pure imperative validity non-conjunctively (see Sect. 4.2).

For a final example of a weakly but not strongly valid argument that one might take to be intuitively invalid, consider the argument — call it *Argument E* — from 'marry him' to 'if you do not marry him, kill him'. Argument E is an instance of what may be called *violating the premisses*, namely a pure imperative argument for which $C'$ entails $V$ (i.e. the context of the conclusion entails the violation proposition of the conjunction of the premisses). It can be shown that every obeyable instance of violating the premisses for which $S'$ is possible is weakly but not strongly valid, but one might claim that many such instances are intuitively invalid. In reply note first that Argument E is also an instance of imperative conjunction elimination: 'marry him' turns out to be the conjunction of 'marry him' with 'if you do not marry him, kill him'. So we are back, *mutatis mutandis*, to a previous discussion; in particular, killing him is conditionally permitted according to the conclusion of Argument E but need not be conditionally permitted simpliciter. Moreover, 'marry him' strongly and intuitively (see n. 40) entails 'if you do not marry him, marry him'; but the latter prescription can also be expressed by 'if you do not marry him, kill him and do not kill him', and the intuition that this entails 'if you do not marry him, kill him' conflicts with the original intuition that Argument E is invalid. Some people, however, might find the original intuition to be much stronger, and might conclude that Argument E is intuitively invalid. For the sake of argument, let me grant this conclusion; after all, my thesis in this subsection is only that *almost* every pure imperative argument that is weakly but not strongly valid is neither intuitively valid nor intuitively invalid. (It can now be seen that 'almost' amounts to 'except for many instances of violating the premisses'.) If Argument E and similar arguments are intuitively invalid, does the fact that they are weakly valid pose a significant problem for

[62] I understand (e.g.) the claim that there is a reason for you to wear a soldier's uniform *given that* you enlist as the claim that there is a (comparative) reason for you to enlist and wear a soldier's uniform *rather than* enlisting and not wearing a soldier's uniform. On arguments similar to Argument D see: Espersen 1967, pp. 76–7; MacKay 1971, pp. 93–4; Rand 1939, p. 318, 1962, p. 248; Ross 1941, p. 67, n. 1; Stolpe 2008, p. 175.

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my definition of weak validity?[63] Some people might think so: Rescher (1966a, pp. 86, 91) and Sosa (1964, pp. 77–8, 1966b, p. 233, 1967, pp. 62–3) seem to consider unacceptable any definition of pure imperative validity that has the consequence that arguments like Argument E are valid. I reply that a similar problem afflicts the standard definition of pure *declarative* validity, which has the consequence that the following arguments are valid (Cooper 1968, pp. 297–8):

(1) You smiled. Therefore, if you did not smile, you killed your daughter.

(2) It is not the case that if she is over eighty she is still young. Therefore, if she is still young she is over eighty.

(3) If John is in Paris, then he is in France. If he is in Berlin, then he is in Germany. Therefore, if John is in Paris he is in Germany, or, if he is in Berlin he is in France.

(4) It is not the case that if my brother is the Pope he is Jewish. Therefore, my brother is the Pope.

(5) There are no dinosaurs. Therefore, every dinosaur can fly, and no dinosaur can fly.

Given that the above arguments are intuitively invalid, the standard definition of pure declarative validity does not fit perfectly our intuitions (cf. Haack 1978, p. 33); but then why demand a perfect fit from a satisfactory definition of pure imperative validity? I propose thus a *principle of parity*: the standards of success for imperative logic should not be higher or lower than those for standard declarative logic (cf. Espersen 1967, p. 62, n. 14; Zellner 1971, pp. 58–9). In accordance with this principle, my goal in this paper is to establish foundations for pure imperative inference that are about *as secure as* the standard foundations of pure declarative inference — not to establish *perfectly* secure foundations. I admit that this defence of my definitions against the charge that they have counterintuitive consequences should be used sparingly; I have used it only once before (in Sect. 5.1), in response to the claim that many unobeyable pure imperative arguments

[63] One might argue that a problem arises with 'contrary-to-duty' imperative inferences: if (1) 'go' entailed (2) 'if you do not go, kill him', then (1), together with (3) 'if you do not go, apologize' and (4) 'you do not go', would entail (5) 'kill him' (via (2) and (4)), whereas (1), (3), and (4) do not entail (5) but rather entail (6) 'apologize'. In reply I deny that the argument from (2) and (4) to (5) is valid; but since this is a *mixed-premiss* imperative argument ((4) is a *proposition*), I defend my view in a sequel to this paper (cf. n. 60).

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are intuitively invalid (despite being strongly valid). Overall, then, my definitions of pure imperative validity fit our intuitions satisfactorily — even if not perfectly. It bears repeating that I am speaking about *tutored* intuitions, and that my approach goes beyond a mere appeal to intuitions: my definitions were motivated by foundational considerations.

## 6. Conclusion

Alf Ross famously asked:

> [W]hat does it mean that $I_2$ is a logical consequence of $I_1$? … I fail to see that a statement of the rules governing the *procedure* by which the transition is made from $I_1$ to $I_2$ contributes to the elucidation of the question: what does such transition mean? … Does the transformation mean anything and more than a word game, a parlour game? (Ross 1941, p. 58)

In this paper I took Ross's questions seriously: rather than just proposing yet another definition of pure imperative validity and trying to defend it primarily by appealing to intuitions concerning the validity of specific pure imperative arguments, I started by inquiring what a definition of pure imperative validity must look like in order to be *useful* (rather than a mere 'parlour game'). This inquiry led me, via general considerations about reasons, to my definitions of strong and weak validity.

It is sometimes suggested in the literature that there is an unbridgeable gap between a satisfaction-based and a bindingness-based approach to pure imperative inference (cf. Hare 1967, p. 325; Lemmon 1965, p. 61; Segerberg 1990, p. 203; Sosa 1970, pp. 223–4). For example, according to Weinberger: 'The satisfaction or non-satisfaction of an imperative bears no relation to its validity [i.e. bindingness]. … There is thus no way to derive an imperative logic out of the satisfaction relation' (1958, p. 30; my translation). The Equivalence Theorem that I proved in this paper provides a way to bridge the gap between the two approaches. Moreover, the equivalence that I proved between weak validity and redundancy-validity provides further evidence for the usefulness of weak validity.

This paper in effect defends a form of *logical pluralism*, understood as 'the view that several … consequence relations have a good claim to be regarded as … providing defensible accounts as to when an argument is (deductively) valid' (Humberstone 2009, p. 162). Indeed,

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I argued that strong and weak validity are both useful, and that pure imperative arguments that are weakly but not strongly valid are neither intuitively valid nor intuitively invalid.[64] On the other hand, the form of logical pluralism that this paper defends is limited: I argued that pure imperative arguments that are strongly (and thus weakly) valid are intuitively valid, and that pure imperative arguments that are not weakly (and thus not strongly) valid are intuitively invalid.

The obvious next step is to generalize my account to *mixed-premiss* imperative arguments, namely arguments whose conclusion is a prescription and whose premisses include both a prescription and a proposition. This is a major task of a sequel to this paper.[65]

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[64] One might wonder why I defend logical pluralism for imperative but not for declarative logic (especially given that, as I said in Sect. 5.3, some intuitively invalid pure declarative arguments are valid according to the standard definition of pure declarative validity). Similarly, one might wonder why I claim that there are three possible satisfaction values for prescriptions (see Vranas 2008, pp. 534–5) but only two possible truth values for propositions. I reply that in this paper I rely on standard declarative logic primarily in order to avoid unnecessary controversy. But I am not denying the possible value of exploring imperative logics based on non-standard declarative logics.

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## Appendix A

## Proof of the Equivalence Theorem

To prove the Equivalence Theorem, I will use a certain assumption. To formulate this assumption, I introduce first a definition: a (comparative) reason is *conditional* on a proposition $P$ exactly if, for all propositions $P_1$ and $P_2$ such that the reason favours $P_1$ over $P_2$, $P_1$ entails $P$ and $P_2$ entails $P$. For example, under appropriate circumstances, the fact that exercising and dieting is better for your health than exercising without dieting is a reason conditional on the proposition that you exercise: it favours some proposition which entails that you exercise over some other such proposition (since it favours the proposition that you exercise and you diet over the proposition that you exercise and you do not diet), but it does not favour any proposition over any other one if it is not the case that both propositions entail that you exercise (e.g. it does not favour the proposition that you exercise and you diet over the proposition that you do not exercise).[66] Given this definition, here is the assumption I will use to prove the Equivalence Theorem:

> Assumption 1: For any given prescription, it is possible for the prescription to be strongly supported by some reason conditional on the context of the prescription exactly if it is possible for the prescription to be obeyed

This assumption captures two ideas. (1) If a prescription is *unobeyable* (i.e. its obedience proposition is — logically — impossible; equivalently, its violation proposition is necessary), like the prescription expressed by 'run and do not run', then there is no possible world in

[66] One might object by claiming that the fact that exercising and dieting is better for your health than exercising without dieting favours the proposition that (1) you plan to exercise and diet over the proposition that (2) you plan to exercise without dieting — although neither of these propositions entails that you exercise. In reply I can stipulate that the 'appropriate circumstances' (under which the above fact is a reason conditional on the proposition that you exercise) include the fact that, given your psychological make-up, if you plan to exercise then you will not exercise (so that the above fact does not favour (1) over (2)).

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which the prescription is strongly binding.[67] (2) On the other hand, if a prescription is *obeyable* (i.e. not unobeyable), then there is a possible world in which the prescription is strongly binding (because it is strongly supported by a reason *conditional* on the context of the prescription). Take, for example, the prescription expressed by 'if you love your children, torture them just for fun'. There is a — maybe extremely 'remote' — possible world in which it is a fact that the expected long-term consequences associated with any (possible) proposition which entails that you love your children and you torture them just for fun are equally good and are better than the expected long-term consequences associated with any different proposition which entails that you love your children and you do not torture them just for fun. In that possible world, the above fact is a reason conditional on the proposition that you love your children and strongly supports the above prescription. One might claim that in such a world the above prescription is not *all-things-considered* binding, or that it is not even pro tanto *morally* binding. But these claims are compatible with Assumption 1: it suffices that the prescription be pro tanto *non*-morally (e.g. prudentially) strongly binding in the above world. So Assumption 1 is weaker than one might think at first sight.

Consider now a pure imperative argument — from now on referred to as 'the Argument' — whose premisses have the prescription $<S, V>$ (namely the ordered pair with first coordinate $S$ and second coordinate $V$) as their conjunction and whose conclusion is $<S', V'>$. My proof of the Equivalence Theorem has four parts.

***First part*:** I will prove that, if $V$ is necessary, or $S'$ entails $S$ and $V'$ entails $V$, then the Argument is strongly valid. (1) If $V$ is necessary, then $<S, V>$ is unobeyable, so by Assumption 1 it is impossible for $<S, V>$ to be strongly supported by any reason (see n. 67); then by Definition 1a the Argument is strongly valid. (2) If $S'$ entails $S$ and $V'$ entails $V$, then (assuming that the above two entailments hold necessarily) it is necessary that every proposition which entails $S'$ also

[67] Assumption 1 entails that it is impossible for an unobeyable prescription to be strongly supported by any reason *conditional* on the context of the prescription; but the context of an unobeyable prescription is necessary, and any reason is conditional on a necessary proposition, so Assumption 1 entails that it is impossible for an unobeyable prescription to be strongly supported by *any* reason (and thus also entails — as one can see by using Definition 4 — that it is impossible for an unobeyable prescription to be *weakly* supported by any reason).

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entails $S$ and every proposition which entails $V'$ also entails $V$, so it is necessary that (a) if a reason $R$ favours every proposition which entails $S$ over every different proposition which entails $V$, then $R$ favours every proposition which entails $S'$ over every different proposition which entails $V'$, and (b) if a reason $R$ does not favour any proposition which entails $S$ over any other such possible proposition, then $R$ does not favour any proposition which entails $S'$ over any other such possible proposition; then by Definition 3 it is necessary that if a reason strongly supports $<S, V>$ it also strongly supports $<S', V'>$, so by Definition 1a the Argument is strongly valid.

***Second part*:** I will prove that, if the Argument is strongly valid, then $V$ is necessary, or $S'$ entails $S$ and $V'$ entails $V$. Consider the contrapositive: if $V$ is not necessary and either $S'$ does not entail $S$ or $V'$ does not entail $V$ (or both), then the Argument is not strongly valid (i.e. by Definition 1a, there is a possible world in which some reason strongly supports $<S, V>$ but does not strongly support $<S', V'>$). To prove this contrapositive, assume that its antecedent holds. Then $V$ is not necessary, so $<S, V>$ is obeyable, and by Assumption 1 there is a possible world $W$ in which some reason $R$ conditional on $C$ strongly supports $<S, V>$. Suppose, for *reductio*, that $R$ in $W$ also strongly supports $<S', V'>$. *First case*: $S'$ does not entail $S$. Then $S' \& \neg S$ (i.e. the conjunction of $S'$ with the negation of $S$) is possible, and is thus different from $S \& V'$ (because the conjunction of $S' \& \neg S$ with $S \& V'$ is impossible, so $S' \& \neg S$ and $S \& V'$ would fail to be different only if they were both impossible). Since (i) $S' \& \neg S$ is different from $S \& V'$, (ii) $S' \& \neg S$ entails $S'$, (iii) $S \& V'$ entails $V'$, and (iv) $R$ in $W$ strongly supports $<S', V'>$ and thus the corresponding dominance condition (see Definition 3) holds, it follows that (1) $R$ in $W$ favours $S' \& \neg S$ over $S \& V'$. Since $R$ is conditional on $C$, it further follows that $S' \& \neg S$ entails $C$. Moreover, $S' \& \neg S$ entails $\neg S$, so it entails $C \& \neg S$, namely $V$. Since (i) $S \& V'$ is different from $S' \& \neg S$, (ii) $S \& V'$ entails $S$, (iii) $S' \& \neg S$ entails $V$, and (iv) $R$ in $W$ strongly supports $<S, V>$ and thus the corresponding dominance condition holds, it follows that (2) $R$ in $W$ favours $S \& V'$ over $S' \& \neg S$. But (1) & (2) contradicts the asymmetry of comparative favouring. *Second case*: $V'$ does not entail $V$. Then $V' \& \neg V$ is possible, and is thus different from $S' \& V$ (because the conjunction of $V' \& \neg V$ with $S' \& V$ is impossible, so $V' \& \neg V$ and $S' \& V$ would fail to be different only if they were both impossible). Since (i) $S' \& V$ is different from $V' \& \neg V$, (ii) $S' \& V$ entails $S'$, (iii) $V' \& \neg V$ entails $V'$, and (iv) $R$ in $W$ strongly supports $<S', V'>$ and

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thus the corresponding dominance condition holds, it follows that (3) $R$ in $W$ favours $S' \& V$ over $V' \& \neg V$. Since $R$ is conditional on $C$, it further follows that $V' \& \neg V$ entails $C$. Moreover, $V' \& \neg V$ entails $\neg V$, so it entails $C \& \neg V$, namely $S$. Since (i) $V' \& \neg V$ is different from $S' \& V$, (ii) $V' \& \neg V$ entails $S$, (iii) $S' \& V$ entails $V$, and (iv) $R$ in $W$ strongly supports $<S, V>$ and thus the corresponding dominance condition holds, it follows that (4) $R$ in $W$ favours $V' \& \neg V$ over $S' \& V$. But (3)&(4) contradicts the asymmetry of comparative favouring, and the *reductio* is complete.

***Third part*:** I will prove that, if $C'$ entails $C$ and $V'$ entails $V$, then the Argument is weakly valid. Suppose that $C'$ entails $C$ and $V'$ entails $V$, and take any possible world $W$ in which some reason $R$ weakly supports $<S, V>$ (if no such possible world exists, then by Definition 1b the Argument is weakly valid). I will prove that $R$ in $W$ also weakly supports $<S', V'>$. By Definition 4, $R$ in $W$ strongly supports some prescription $I^*$ whose satisfaction proposition $S^*$ entails $S$ and whose context is $C$. Consider the prescription $I'' = <S^* \& S', C' \& \neg(S^* \& S')>$. Its violation proposition can be shown to entail the violation proposition of $I^*$,[68] and $S^* \& S'$ entails $S^*$, so by what I proved in the first part above the argument from $I^*$ to $I''$ is strongly valid, and by Definition 1a $R$ in $W$ strongly supports $I''$. But the satisfaction proposition of $I''$ entails $S'$ and the context of $I''$ is $C'$, so by Definition 4 $R$ in $W$ weakly supports $<S', V'>$.

***Fourth part*:** I will prove that, if the Argument is weakly valid, then $C'$ entails $C$ and $V'$ entails $V$. Consider the contrapositive: if either $C'$ does not entail $C$ or $V'$ does not entail $V$ (or both), then the Argument is not weakly valid (i.e. by Definition 1b, there is a possible world in which some reason weakly supports $<S, V>$ but does not weakly support $<S', V'>$). To prove this contrapositive, assume that its antecedent holds. Then $V$ is not necessary (because if $V$ and thus also $C$ were necessary, then $C'$ would entail $C$ and $V'$ would entail $V$), so $<S, V>$ is obeyable, and by Assumption 1 there is a possible world $W$ in which some reason $R$ conditional on $C$ strongly, and thus also weakly, supports $<S, V>$. Suppose, for *reductio*, that $R$ in $W$ also

[68] The violation proposition of $I''$, namely $C' \& \neg(S^* \& S')$, is equivalent to $(C' \& \neg S^*) \vee (C' \& \neg S')$. Since $C'$ entails $C$, $C' \& \neg S^*$ entails $C \& \neg S^*$. Moreover, $C' \& \neg S'$ entails $C \& \neg S^*$ (because $C' \& \neg S'$, which is $V'$, entails $V$, which is $C \& \neg S$, and which entails $C \& \neg S^*$ because $S^*$ entails $S$). So $(C' \& \neg S^*) \vee (C' \& \neg S')$ entails $C \& \neg S^*$, which is the violation proposition of $I^*$.

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weakly supports $<S', V'>$. Then, by Definition 4, $R$ in $W$ strongly supports some prescription $I^*$ whose satisfaction proposition $S^*$ entails $S'$ and whose context is $C'$ — and thus whose violation proposition $V^*$ is entailed by $V'$. Since $R$ in $W$ strongly supports $<S^*, V^*>$, $R$ in $W$ favours $S^*$ over $V^*$. Since $R$ is conditional on $C$, both $S^*$ and $V^*$ entail $C$, and thus so does their disjunction, namely $C'$. Since $C'$ entails $C$ but either $C'$ does not entail $C$ or $V'$ does not entail $V$ (or both), $V'$ does not entail $V$. Then $V' \& \neg V$ is possible, and is thus different from $S^* \& V$ (because the conjunction of $V' \& \neg V$ with $S^* \& V$ is impossible, so $V' \& \neg V$ and $S^* \& V$ would fail to be different only if they were both impossible). Since (i) $S^* \& V$ is different from $V' \& \neg V$, (ii) $S^* \& V$ entails $S^*$, (iii) $V' \& \neg V$ entails $V^*$ (because $V' \& \neg V$ entails $V'$, which entails $V^*$), and (iv) $R$ in $W$ strongly supports $<S^*, V^*>$ and thus the corresponding dominance condition holds, it follows that (1) $R$ in $W$ favours $S^* \& V$ over $V' \& \neg V$. Since $R$ is conditional on $C$, it further follows that $V' \& \neg V$ entails $C$. Moreover, $V' \& \neg V$ entails $\neg V$, so it entails $C \& \neg V$, namely $S$. Since (i) $V' \& \neg V$ is different from $S^* \& V$, (ii) $V' \& \neg V$ entails $S$, (iii) $S^* \& V$ entails $V$, and (iv) $R$ in $W$ strongly supports $<S, V>$ and thus the corresponding dominance condition holds, it follows that (2) $R$ in $W$ favours $V' \& \neg V$ over $S^* \& V$. But (1) & (2) contradicts the asymmetry of comparative favouring, and the *reductio* is complete.

(Note that satisfaction indifference appears only in the first of the above four parts. So I can now explain my remark, at the end of Sect. 3.2, that for my purposes in this paper it does not really matter whether one includes satisfaction indifference or violation indifference — or both, or neither — in a definition of strong support: it can be seen that all corresponding versions of the Equivalence Theorem can be proved in essentially the same four-part way, with only the first part requiring some — minor — modifications.)

## Appendix B

### Alternative definitions of pure imperative validity

In this appendix I argue that my definitions of pure imperative validity are preferable to all alternative definitions that have been proposed in the literature and to some alternative definitions that might be proposed. I use five (groups of) criteria in evaluating or comparing definitions: (1) whether the definitions are *formally acceptable*, in the

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sense of satisfying reflexivity and transitivity,[69] (2) whether the definitions are *intuitively acceptable*, in the sense of fitting our (tutored) intuitions, (3) whether the definitions are *principled*, in the sense of being motivated by considerations that go beyond a mere appeal to intuitions, (4) whether the definitions are *usable*, in the sense of enabling one to decide whether specific pure imperative arguments are valid, and (5) whether the definitions are *useful*, in the sense of having (D1)–(D5) (Sect. 2), (D6) (Sect. 3.4), and (D7) (Sect. 4.2) as consequences. Given that any definition that has (D7) as a consequence also has the consequence that the validity of a multiple-premiss pure imperative argument amounts to the validity of the corresponding single-premiss argument, to simplify the exposition I formulate most of the definitions that follow only for single-premiss pure imperative arguments.

### *B.1 Five reductive definitions*

Say that a definition of pure imperative validity is *reductive* exactly if it reduces the validity of a pure imperative argument to the (standard) validity of a corresponding pure declarative argument. Some reductive definitions can be formulated as follows (for single-premiss arguments): 'the pure imperative argument from $I$ to $I'$ is valid exactly if the (corresponding) pure declarative argument is valid whose premiss and conclusion are respectively the propositions that correspond to

[69] Besides reflexivity and transitivity, the following conditions might be considered necessary for formal acceptability (cf. Alchourrón and Martino 1990, pp. 57–8; Koslow 1992, p. 5, 1999, p. 113; Westerhoff 2005, p. 615). (Notation: $\Gamma$ and $\Delta$ are sets of prescriptions ($\Gamma$ is non-empty), and $I$ and $I'$ are prescriptions.) (1) *Weak cut*: if $\Gamma$ entails $I$ and $\Gamma\cup\{I\}$ entails $I'$, then $\Gamma$ entails $I'$. (2) *Strong cut*: if $\Gamma$ entails $I$ and $\Delta\cup\{I\}$ entails $I'$, then $\Gamma\cup\Delta$ entails $I'$. (3) *Idempotence*: $\Gamma$ entails $I$ exactly if the set of all and only those prescriptions that are entailed by $\Gamma$ entails $I$. (4) *Projection*: if $I\in\Gamma$, then $\Gamma$ entails $I$. (5) *Monotonicity*: if $\Gamma$ entails $I$ and $\Gamma\subseteq\Delta$, then $\Delta$ entails $I$. It can be shown that (a) transitivity (namely: if $\Gamma$ entails $I$ and $\{I\}$ entails $I'$, then $\Gamma$ entails $I'$) follows from strong cut, from weak cut plus monotonicity, and from idempotence plus monotonicity, (b) weak cut follows from strong cut and from idempotence plus monotonicity and projection, (c) reflexivity (namely: $\{I\}$ entails $I$) follows from projection, (d) projection follows from monotonicity plus reflexivity, and (e) monotonicity follows from strong cut plus projection. It can also be shown that weak validity (strictly speaking: weak entailment) satisfies all of the above conditions, and that strong validity satisfies weak cut and idempotence but not strong cut, projection, or monotonicity. Geach (1966, p. 77) and Kenny (1975, pp. 92–5, 1989, pp. 44–5) reject monotonicity but MacKay (1971, p. 92) accepts it; I submit that conflicting intuitions about monotonicity are largely explained by a vacillation between strong and weak validity. (I do not consider conditions (1)–(5) necessary for formal acceptability because, unlike reflexivity and transitivity, conditions (1)–(5) are not important for usefulness; so strictly speaking I subsume formal acceptability under usefulness.)

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*I* and to *I′*' (cf. Dubislav 1937, p. 341). I will examine the reductive definitions according to which the proposition that corresponds to a given prescription is (1) the satisfaction proposition *S* of the prescription (satisfaction-validity), (2) the obedience proposition $\neg V$ of the prescription (obedience-validity), (3) the material conditional $C \rightarrow S$, where *C* is the context of the prescription (conditional-satisfaction-validity), (4) the material conditional $V \rightarrow P$, where *P* is a proposition specifying a 'penalty' associated with the violation of the prescription (penalty-validity), and (5) the proposition that the prescription is binding (bindingness-validity).[70]

*B.1.1 Satisfaction-validity*

The argument from *I* to *I′* is satisfaction-valid exactly if *S* entails *S′*. This definition is usable and formally acceptable, but is not principled and is arguably not useful (arguably it does not have any of (D1)–(D6) as a consequence[71]). Nor is it intuitively acceptable: the argument from 'if it rains, close the window' to 'if it rains and thunders, close the window' (Sect. 5.1) is intuitively valid but satisfaction-invalid (and similarly for many other instances of strengthening the antecedent), and the argument from 'if you wake up and you see a burglar, call the police' to 'if you wake up, call the police' (Sect. 5.2.3) is intuitively invalid but satisfaction-valid (and similarly for many other instances of extending the context).

[70] (1) On satisfaction-validity see: Aloni 2003, p. 61; Bennett 1970, p. 314; Bergström 1962, pp. 37–42; Black 1964, p. 168; Boisvert and Ludwig 2006, p. 882; Castañeda 1960a, p. 27; Clarke 1973, p. 193, 1975, pp. 417–18, 1985, p. 101; Clarke and Behling 1998, p. 283; Fisher 1962, pp. 232–3; Gensler 1996, pp. 183–4; Hare 1967, pp. 324–5; Harnish 2006; Hofstadter and McKinsey 1939, p. 452; Katz 1977, pp. 229–30; Niiniluoto 1985, p. 177; Rescher 1966a, pp. 89–91; Ross 1941, pp. 60–2; Vanderveken 1990, p. 52; Wedeking 1969, p. 45. Cf.: Belnap 1966; Binkley 1966; Gibbard 2003, pp. 46–7; Simon 1965, 1966a, 1966b. (2) On obedience-validity see: Rescher 1966a, pp. 90–1. (3) On penalty-validity see: Bohnert 1945; Espersen 1967, p. 79; Fitch 1946; Fulda 1995; Gensler 1996, pp. 184–5; Menger 1939, p. 59; Moutafakis 1975, pp. 17, 62–7. (4) On bindingness-validity see: Aune 1977, p. 176; Bennett 1970, p. 315; Castañeda 1960a, p. 43, 1960b, p. 155, 1974, chap. 4, 1975, pp. 121–3; Chaturvedi 1980, p. 474; Espersen 1967, pp. 77–8; Frey 1957, p. 465; Green 1998, p. 719; Grue-Sörensen 1939, pp. 197–8; Hare 1967, pp. 324–5; Kapitan 1984; Lemmon 1965, p. 61; Mitchell 1990, pp. 485–97; Moritz 1941, pp. 240–1; Raz 1977, p. 83; Ross 1941, pp. 58–60; Sosa 1967, pp. 60–2; Wedeking 1969, pp. 102–14; Zellner 1971, pp. 49–51.

[71] Concerning (D1): since some satisfaction-valid arguments are neither weakly nor strongly valid, it is possible that some reason *R* supports (weakly or strongly) the premiss but not the conclusion of a satisfaction-valid argument, so if *R* is complete (see n. 10) and assumption (A1) of n. 10 holds, then by reasoning as in that note one can show that it is possible that *R* supports the premiss but no complete reason supports the conclusion of a satisfaction-valid argument. Similarly concerning (D2).

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*B.1.2 Obedience-validity*

The argument from $I$ to $I'$ is obedience-valid exactly if $\neg V$ entails $\neg V'$ (i.e. $V'$ entails $V$). (For unconditional prescriptions, obedience-validity amounts to satisfaction-validity.) This definition is usable and formally acceptable, but is not principled and is arguably not useful (arguably it does not have any of (D1)–(D6) as a consequence). Nor is it intuitively acceptable: the arguments from 'if the volcano erupts, flee' to 'if you do not flee, let it not be the case that the volcano erupts' (Sect. 5.2.2) and from 'if you answer the first question, do not answer both questions' to 'do not answer both questions' (cf. Sect. 5.2.3) are intuitively invalid but obedience-valid (and similarly for many other instances of contraposition and of extending the context).[72]

*B.1.3 Conditional-satisfaction-validity*

The argument from $I$ to $I'$ is conditional-satisfaction-valid exactly if $C \rightarrow S$ entails $C' \rightarrow S'$. Surprisingly, this kind of validity (which captures the thought that pure imperative validity is isomorphic to pure declarative validity) amounts to obedience-validity: $C \rightarrow S$ entails $C' \rightarrow S'$ exactly if $\neg(C \& \neg S)$ entails $\neg(C' \& \neg S')$, namely exactly if $\neg V$ entails $\neg V'$.

*B.1.4 Penalty-validity*

The argument from $I$ to $I'$ is penalty-valid exactly if $V \rightarrow P$ entails $V' \rightarrow P'$. This definition is formally acceptable, but is not principled and is arguably not useful (arguably it does not have any of (D1)–(D6) as a consequence). Nor is it usable: it is not clear what the penalty $P$ associated with (the violation of) any given prescription is (cf. Fulda 1995, pp. 7–8). One might respond that the definition is *partly* usable, for two reasons. First, because in many cases it is clear that $P = P'$; for example, this is so in some cases in which $I$ and $I'$ are both only about answering questions on a pass-fail exam (so the penalty would be failure). Second, because the penalty associated with certain

[72] Another intuitively invalid but obedience-valid argument is the argument from 'if you answer the first question, do not answer all three questions' to 'if you answer the second question, do not answer all three questions'. Moreover, every instance of violating the premisses (Sect. 5.3) is obedience-valid (although this is irrelevant to a comparison of obedience- with weak validity). On the other hand, every satisfiable instance of violating the premisses is satisfaction-invalid; but even if one takes this to confer an advantage (in terms of intuitive acceptability) on satisfaction- over weak validity, this advantage is offset by the counterintuitive consequences of the definition of satisfaction-validity (Sect. B.1.1).

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prescriptions is 'null', namely a tautology; for example, Fulda (1995, p. 7) claims that this is so for 'excuse me'. But then, I reply, the definition is not intuitively acceptable, for two reasons. First, because every obedience-valid argument for which $P = P'$ is penalty-valid (if $P = P'$, then it turns out that $V \rightarrow P$ entails $V' \rightarrow P$ exactly if $V'$ entails $V \vee P$, and thus if $V'$ entails $V$), but some obedience-valid arguments for which (proponents of penalty-validity would say) it is clear that $P = P'$ are intuitively invalid (Sect. B.1.2). Second, because if the penalty associated with 'excuse me' is a tautology, then for *every* prescription $I$ (e.g. 'do not excuse me') the argument from $I$ to 'excuse me' is penalty-valid.

*B.1.5 Bindingness-validity*

The argument from $I$ to $I'$ is bindingness-valid exactly if the proposition that $I$ is binding entails the proposition that $I'$ is binding. This definition is formally acceptable and principled, but is not in any obvious way usable, so it is not clear how to assess its intuitive acceptability. It turns out, however, that *if* a certain assumption holds, then a variant of this definition is *equivalent* to a corresponding variant of Definition 1 (see n. 10 for details), and then bindingness-validity is after all usable but is no real rival to strong or weak validity.

*B.2 Five further definitions*

*B.2.1 Assent-validity*

According to Hare: 'A sentence P entails a sentence Q if and only if the fact that a person assents to P but dissents from Q is a sufficient condition for saying he has misunderstood one or other of the sentences' (1952, p. 25). To this definition one might object that a person who fails to realize that P entails Q can assent to P and dissent from Q despite understanding both P and Q (cf. Kelsen 1979, p. 336, 1991, p. 401).[73] Moreover, Hare's definition is about (imperative and declarative) *sentences*, not directly about *prescriptions*. To avoid these problems, say that the argument from $I$ to $I'$ is *assent-valid* exactly

[73] See Castañeda 1960a, p. 30 for a possible reply. On Hare's definition see also: Bennett 1970, p. 318; Bhat 1983; Castañeda 1960a, pp. 28–33; Clarke 1985, pp. 5, 103; Duncan-Jones 1952; Espersen 1967, pp. 67–8; Hansen 2008, pp. 36–8; Hare 1952, p. 172, 1967, pp. 325–6, 1995; Hoche 1995a, pp. 230–2, 1995b, pp. 339–40; Keene 1966, pp. 62–3; MacKay 1969; Moutafakis 1975, pp. 71–8; Wedeking 1969, pp. 100–2; Zellner 1971, pp. 25–33; cf. Gombay 1967, pp. 150–1; Hare 1949, pp. 31–7; Stalley 1972. On related definitions see: Binkley 1965, pp. 436–48 (cf. Aune 1977, pp. 158–67); McArthur and Welker 1974; Searle and Vanderveken 1985; Vanderveken 1990.

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if, necessarily, every person who assents (or is committed to assenting) to $I$ is (thereby) *committed* to assenting to $I'$ (cf. Hare 1969, p. 65). (Alternatively, following Rescher (1966a, pp. 77–8), one might say that the argument from $I$ to $I'$ is assent-valid exactly if, necessarily, every person who — at least implicitly — assents to $I$ also *at least implicitly* assents to $I'$.) Say also that a person *assents* to (or *endorses*) a prescription exactly if the person takes the prescription to be binding (cf. Gauthier 1963, pp. 63–4; Chaturvedi 1980, p. 474). (Alternatively, following Hare (1952, pp. 19–20, 1969, p. 66), one might define *assenting* to a prescription as prescribing that the prescription be satisfied or agreeing to satisfy it.) This definition of pure imperative validity is formally acceptable and principled, but I take its major flaw to be that it is not usable; for example, it is not clear whether it is necessary that every person who assents (or is committed to assenting) to 'Smith, open the window' is committed to assenting (or at least implicitly assents) to 'Smith, if the sun shines, open the window'. So it is also not clear how to assess the intuitive acceptability of the definition.

*B.2.2 Satisfactoriness-validity*

Following Kenny (1966, pp. 71–4, 1975, pp. 80–2), say that a prescription is *satisfactory* relative to a given set of goals exactly if the satisfaction proposition of the prescription entails that every goal in the set is achieved, and say that the argument from $I$ to $I'$ is *satisfactoriness-valid* exactly if, necessarily, for every set of goals, if $I$ is satisfactory relative to that set, then $I'$ is also satisfactory relative to that set. (Satisficing validity (Sect. 3.4), to which the following discussion also applies, amounts to satisfactoriness-validity if $I$ and $I'$ are unconditional.) This definition of pure imperative validity is formally acceptable and principled. Moreover, it can be shown (see Kenny 1966, p. 74, 1975, p. 82) that the argument from $I$ to $I'$ is satisfactoriness-valid exactly if the 'converse' argument from $I'$ to $I$ is satisfaction-valid (i.e. exactly if $S'$ entails $S$), so the definition is also usable. The definition is not useful, however: it does not have (D6) as a consequence (Sect. 3.4). Nor is it intuitively acceptable: the arguments from 'open the door' to 'open the door and smash the window' (Kenny 1966, p. 74, 1975, p. 83), from 'bring me a drink' to 'bring me a poisoned drink' (Gombay 1967, p. 146), and from 'read at least one book' to 'read a million books' are intuitively invalid but satisfactoriness-valid. Proponents of satisfactoriness-validity might respond: 'a man who is told to open the door, and reasons that a *sufficient* condition of this would be to open the door and smash the

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window, is reasoning with perfect logic' (Hare 1969, p. 67). In reply I can grant this, but it does not follow that the argument from 'open the door' to 'open the door and smash the window' is intuitively valid; to use an analogy, someone who is told that the door is open, and reasons that a sufficient condition for this being true is that the door is open and the window is smashed, may be reasoning with perfect logic, but it does not follow that the argument from 'the door is open' to 'the door is open and the window is smashed' is intuitively valid. Proponents of satisfactoriness-validity might also argue that, by opening the door and smashing the window, 'the agent would no doubt annoy the commander; but this would be because he would be acting against the commander's tacit desire that the window should not be broken' (Kenny 1966, p. 74, 1975, p. 83). 'If my *only* want is to have the door open, why should I object if someone opens the door and smashes the window?' (Kenny 1975, p. 91.) I reply that the appeal to tacit desires — or to 'bad consequences' (Raz 1978b, p. 11) — is a red herring: if from 'open the door' someone infers 'open the door and cure cancer', and proceeds to do both, I will not object to her curing cancer, but I will still object to her *inference*.[74]

*B.2.3 Rescher's definition*

According to Rescher (1966a, pp. 82–91), the pure imperative argument from $I_1, \ldots, I_n$ to $I'$ is valid exactly if, letting $S^*$ be the conjunction of the satisfaction propositions of the premisses, $S^* \& V'$ is impossible and, if $S^*$ is possible, then $S^* \& S'$ is also possible. This definition is usable, but is not principled and is arguably not useful (it does not have (D7) as a consequence: as one can show by using Definition 6, $S^*$ in general differs from the satisfaction proposition of

[74] Strictly speaking, Kenny claims that 'the *command* "open the door and smash the window" can't be inferred from the *command* "open the door": the logic of satisfactoriness concerns fiats, not directives' (1966, p. 74, 1975, p. 83), and grants that 'the logic of satisfactoriness needs very drastic supplementation if it is to be regarded as the logic of commands' (1975, p. 91; cf. 1975, p. 84, n. 11). On satisfactoriness-validity see also: Anscombe 1989, pp. 385–6, 1995, pp. 12–3; Aune 1977, pp. 125–31; Bennett 1970, pp. 316–17; Clarke 1979b, pp. 280–1; Geach 1966; Gombay 1967; Hansen 2008, p. 34; Hare 1967, pp. 324–5, 1969, pp. 64–8; Kenny 1989, pp. 43–5; Milo 1976, p. 16; Mitchell 1990, pp. 480–6; Raz 1978b, pp. 9–11; Ross 1968, pp. 176–7; Sosa 1966a, pp. 215–23; Zellner 1971, pp. 57–8. Note finally that satisficing validity (as opposed to satisfactoriness-validity) is intuitively unacceptable also because the argument from 'if it rains, close the window' to 'if it rains and thunders, close the window' (Sect. 5.1) is intuitively valid but is not satisficingly valid (and similarly for many other instances of strengthening the antecedent).

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the conjunction of the premisses). Nor is it formally acceptable: it violates transitivity (Sosa 1967, p. 63; cf. Chellas 1969, p. 101, n. 14; Espersen 1967, p. 76; Green 1998, p. 720). Finally, it is not intuitively acceptable: the argument from 'answer both questions' to 'if you do not answer the first question, answer both questions' is intuitively valid (cf. Sect. 5.1) but is invalid (cf. Chellas 1969, pp. 100–1) according to Rescher's definition (and similarly for many other instances of strengthening the antecedent), and the arguments from 'if you wake up and you see a burglar, call the police' to 'if you wake up, call the police' (Sect. 5.2.3; cf. Castañeda 1970, p. 443) and from 'if you love him, marry him' and 'if you do not love him, do not marry him' to 'kill him' (cf. Castañeda 1970, pp. 442–3; Zellner 1971, pp. 59–61) are intuitively invalid but are valid according to Rescher's definition (and similarly for many other instances of extending the context and arguments with jointly unsatisfiable premisses).

*B.2.4 Sosa's definition*

According to Sosa (1966b, p. 232, 1967, p. 62, 1970, p. 216; cf. 1964, pp. 71, 77), a pure imperative argument with conclusion $I'$ is valid exactly if some of its premisses are such that (1) the conjunction of their satisfaction propositions is possible and entails $S'$, and (2) the disjunction of their violation propositions is entailed by $V'$. This definition is usable, but is not principled and is arguably not useful (it does not have (D7) as a consequence: see Sect. B.2.3). Nor is it formally acceptable: it violates reflexivity (e.g. the argument from 'run and do not run' to 'run and do not run' is invalid according to Sosa's definition). Finally, it is not intuitively acceptable: the argument from 'if you answer the first question, do not answer both questions' to 'do not answer both questions' (cf. Sect. 5.2.3) is intuitively invalid but is valid according to Sosa's definition (and similarly for many other instances of extending the context), and the arguments from 'if it rains, close the window' to 'if it rains and thunders, close the window' (see n. 39) and from 'if you love me, kiss me' and 'if you do not love me, kiss me' to 'kiss me' are intuitively valid but are invalid according to Sosa's definition (and similarly for many other instances of strengthening the antecedent and arguments with jointly unsatisfiable premisses).

*B.2.5 Semi-inconsistency-validity*

Say that the argument from $I$ to $I'$ is *inconsistency-valid* exactly if $I$ and the negation of $I'$ are inconsistent (cf. Geach 1963, p. 38; Gensler 1990,

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p. 193, 1996, p. 183; Hare 1969, pp. 72–3, 1995, p. 276; Lemmon 1965, p. 55; Moser 1956, p. 200; Wedeking 1969, pp. 157–8). In a previous paper (Vranas 2008, pp. 536–8, 545–8) I have defended the following definitions: (1) the *negation* of the prescription $<S, V>$ is the prescription $<V, S>$, and (2) two prescriptions are *inconsistent* exactly if the disjunction of their violation propositions is necessary. But then the above definition of pure imperative validity violates reflexivity: the argument from 'if it rains, run' to 'if it rains, run' is not inconsistency-valid because the prescriptions 'if it rains, run' and 'if it rains, do not run' (the latter being the negation of the former) are not inconsistent (the disjunction of their violation propositions, namely of the propositions that it rains and you do not run and that it rains and you run, is not necessary).[75] In response one might claim that 'if it rains, run' and 'if it rains, do not run' should count as inconsistent. I disagree (Vranas 2008, p. 547), but let us call these two prescriptions *semi-inconsistent*. More generally, say that two prescriptions are semi-inconsistent exactly if their conjunction is unsatisfiable, and say that the argument from $I$ to $I'$ is *semi-inconsistency-valid* exactly if $I$ and the negation of $I'$ are semi-inconsistent; equivalently (as one can show by using Definition 6), exactly if $S$ entails $S'$ and $V'$ entails $V$. (So a pure imperative argument is semi-inconsistency-valid exactly if it is *both* satisfaction-valid and obedience-valid. Cf. Espersen 1967, pp. 74–5.) This definition of pure imperative validity is principled, formally acceptable, and usable, but is arguably not useful (arguably it does not have any of (D1)–(D6) as a consequence). Nor is it intuitively acceptable: the argument from 'if you answer the first question, do not answer both questions' to 'do not answer both questions' (cf. Sect. 5.2.3) is intuitively invalid but semi-inconsistency-valid (and similarly for many other instances of extending the context), and the argument from 'if it rains, close the window' to 'if it rains and thunders, close the window' (Sect. 5.1) is intuitively valid but semi-inconsistency-invalid

[75] One might conjecture that, if the prescriptions $I$ and $I'$ are inconsistent, then either the argument from $I$ to the negation of $I'$ or the argument from $I'$ to the negation of $I$ is intuitively valid. This conjecture is false, however, as one can check by considering for example the inconsistent prescriptions 'if you sing or dance, do not sing' and 'if you do not sing or do not dance, sing'.

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(and similarly for many other instances of strengthening the antecedent).[76]

[76] Let me also briefly examine some additional approaches to pure imperative inference. (1) Makinson and van der Torre's (2000, 2001, 2003a, 2003b, 2007) 'input/output logics' can be understood as specifying when an ordered pair of sentences follows from a set of such pairs; so if the ordered pair $<A, B>$ expresses (the prescription expressed by) 'if $A$ is true, let $B$ be true', then input/output logics might be understood as providing definitions of pure imperative validity. But in input/output logics (contrary to my approach) two ordered pairs $<A, B>$ and $<A', B'>$ can express (what I take to be) the same prescription even if $B$ and $B'$ are not logically equivalent. For example, if $R$ = 'you run' and $S$ = 'you smile', then $<R, S>$ and $<R, R\&S>$ both express 'if you run, smile', but $S$ and $R\&S$ are not logically equivalent. So reflexivity is not automatically satisfied: 'if you run, smile' need not follow from itself, because $<R, R\&S>$ need not follow from $<R, S>$ in input/output logics. In fact, it turns out that reflexivity is satisfied only in input/output logics (among those examined in Makinson and van der Torre 2000) in which $<A, A>$ may be inferred from any premisses. But as I argued in Sect. 5.2.3, it is not the case that an unviolable prescription (like 'if you murder, murder') follows from every prescription. (2) Based on Veltman's (1996) update semantics, Mastop (2005) proposes a definition of (pure) imperative validity which has the counterintuitive consequences that (a) the argument from 'if it rains, close the window' to 'if it rains and thunders, close the window' is invalid and (b) from 'post the letter or burn it' and 'do not burn the letter' (in this order *but not in the reverse order*) *every* prescription follows (2005, pp. 105–6). (Mastop's motivation for (b) is to avoid Ross's paradox (n. 52), but — contrary to what he claims (2005, p. 107) — on his definition the argument from 'post the letter' to 'post the letter or burn it' is *valid*.) (3) Chellas (1969, 1971) proposes a definition of (pure) imperative validity which has the counterintuitive consequence that arguments expressed by 'do X or Y; so do X or do Y' are in general invalid (1969, p. 23). (A definition proposed by Segerberg (1990, pp. 215–16) also has this counterintuitive consequence.) Moreover, Chellas takes prescriptions to be two-valued (1969, pp. 3, 38, 1971, pp. 116–17), but in a previous paper (Vranas 2008, pp. 534–5) I have argued that they are three-valued. (4) Świrydowicz (1994) proposes a definition of (pure) imperative validity which has the counterintuitive consequence that the — strongly valid, and indeed trivial (Sect. 5) — argument from 'kiss me' and 'if you kiss me, hug me' to 'kiss me and hug me' is invalid.

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