Imperatives, Logic of
Peter B. M. Vranas

Suppose that a sign at the entrance of a restaurant reads: “Do not enter these premises unless you have a reservation and you are properly attired.” You see someone who is properly attired and is about to enter, and you tell her: “Don't enter these premises if you don't have a reservation.” She asks why, and you reply: “It follows from what the sign says.” It seems that you made a valid inference from an imperative premise to an imperative conclusion. But it also seems that imperatives cannot be true or false; so in what sense is your inference valid? Its validity cannot consist in the truth of its premise guaranteeing the truth of its conclusion. One is thus faced with “Jørgensen's dilemma” (Ross 1941: 55–6): apparently, imperative logic cannot exist, because logic deals only with entities that, unlike imperatives, can be true or false; but apparently, imperative logic must exist. It must exist not only because inferences with imperatives can be valid, but also because imperatives (like “Enter” and “Don't enter”) can be inconsistent with each other, and because one can apply logical operations to imperatives: “Don't enter” is the negation of “Enter,” and “Sing or dance” is the disjunction of “Sing” and “Dance.” A standard reaction to this dilemma consists in basing imperative logic on analogues of truth and falsity. For example, the imperative “Don't enter” is satisfied if you don't enter and is violated if you enter, and one might say that an inference from an imperative premise to an imperative conclusion is valid exactly if the satisfaction (rather than the truth) of the premise guarantees the satisfaction of the conclusion. Imperative logic may matter for metaethics (see metaethics): if one understands moral claims as disguised imperatives (see prescriptivism) and thus as lacking truth values (see non-cognitivism), then imperative logic may help explain how inferences involving moral claims can be valid (see frege–geach objection). But before getting into the details of imperative logic, more needs to be said about what exactly imperatives are.

The Nature of Imperatives

The word “statement” is arguably ambiguous between designating a declarative sentence and designating what (an utterance of) such a sentence typically expresses, namely a proposition. Similarly, the word “imperative” is arguably ambiguous between designating an imperative sentence and designating what (an utterance of) such a sentence typically expresses, namely a prescription. Some people deny that propositions exist; similarly, some people may deny that prescriptions exist. But if one accepts that propositions and prescriptions exist, the following is natural to say. A proposition can be expressed by performing the speech acts of asserting, conjecturing, admitting, predicting, and so on. Similarly, a prescription can be expressed...
by performing the speech acts of commanding, advising, requesting, suggesting, and so on. Distinct imperative sentences, for example the English sentence “Enter” and the French sentence “Entrez,” can express the same prescription. A prescription can be expressed by a declarative sentence (for example, “You will now leave”), and a proposition can be expressed by an imperative sentence (for example, “Marry in haste and repent at leisure”). Propositions are the primary – and declarative sentences are secondary – bearers of the semantic values of truth and falsity; similarly, prescriptions are the primary – and imperative sentences are secondary – bearers of whatever semantic values they have. Now three questions arise. (1) What are the semantic values of prescriptions? (2) What kinds of prescriptions are there? (3) How are prescriptions to be modeled for the purposes of logic? The rest of this section considers these three questions in order.

**The semantic values of prescriptions**

There is widespread – though not universal – agreement that prescriptions cannot be true or false (for references, see Vranas 2008: 552). Two main analogues of truth have been proposed for prescriptions (and for imperative sentences): bindingness and satisfaction. The prescription expressed by (addressing to you the imperative sentence) “Go” is – all things considered – binding exactly if there is some – undefeated – reason for you to go; it is satisfied exactly if you go, and violated exactly if you don’t go. The prescription expressed by “If it rains, run” is satisfied if it rains and you run, is violated if it rains and you don’t run, and is avoided (neither satisfied nor violated) if it doesn’t rain. (Some people deny that prescriptions can ever be avoided, and they would say that the prescription expressed by “If it rains, run” is satisfied if it doesn’t rain [see Dummett 1959: 150–1].) One might want to distinguish satisfaction from intentional satisfaction by saying, for example, that if you get out not because I ordered you to get out (suppose you did not hear me), but rather because you wanted to get out anyway, then my order is satisfied but not intentionally satisfied. But the imperative sentence “Get out” might express a prescription that is satisfied exactly if you get out, no matter for what reason (and thus is satisfied in the above example); and the sentence might also express a prescription that is satisfied exactly if you get out because I order you to do so (and thus is not satisfied in the above example). So there is no need to invoke a distinction between satisfaction and intentional satisfaction in response to the above example; one can instead invoke a distinction between two prescriptions that the imperative sentence “Get out” might express (see Vranas 2008: 534).

**Kinds of prescriptions**

Call a prescription (or an imperative sentence) personal exactly if its satisfaction requires the performance of an action, and call it impersonal otherwise. For example, the prescription expressed by “Someone turn off the light” is personal – it is satisfied only if someone performs the action of turning off the light; but the prescription expressed by “Let there be no light in the room” is impersonal – it is satisfied even if
no one performs any action but the room goes dark due to a power failure. (Personal imperative sentences are also known as directives, and impersonal imperative sentences are also known as fiats [see Hofstadter and McKinsey 1939: 446]. Some people deny that fiats exist.) Unlike “Someone turn off the light,” some personal prescriptions are satisfied only if a particular agent or group of agents performs an action; for example, “Max, turn off the light.” Some people (for references, see Vranas 2008: 555) deny the existence of unsatisfiable prescriptions (“Disprove the Pythagorean theorem”), of unviolable prescriptions (“Don’t disprove the Pythagorean theorem”), or of prescriptions about the past (“Open the door yesterday”).

**Models of prescriptions**

Some people propose identifying prescriptions with propositions (or imperative sentences with declarative ones), but such proposals are subject to numerous objections (Hamblin 1987: 97–136). For example, one reason why “Run” cannot be identified with the deontic proposition (or sentence) “You must run” – and thus one reason why imperative logic differs from deontic logic – is that “You must run,” unlike “Run,” entails that there is a (normative) reason for you to run (see DEONTIC LOGIC). (This is not to say that whenever someone – or the law – says “You must run” there is a reason for you to run; it is rather to say that, necessarily, if it is true that you must run, then there is a reason for you to run. Still, some people deny the entailment.)

Other people propose modeling prescriptions (or imperative sentences) in terms of two factors: a factor – sometimes called a mood indicator – indicating that something is being prescribed, and a factor – sometimes called a sentence radical – indicating what is being prescribed (see Hare 1952; Stenius 1967). An objection to such proposals is that they do not adequately model conditional prescriptions: it seems that “If it rains, run” should be modeled in terms of two “sentence radicals,” one corresponding to “It rains” and one corresponding to “Run.” (This is because the prescription “If it rains, run” is avoided if it doesn’t rain, but a prescription modeled in terms of a single sentence radical is never avoided: it is satisfied or violated, depending on whether the sentence radical is true or false.)

Vranas (2008) models a prescription as an ordered pair of logically incompatible propositions, namely the satisfaction proposition and the violation proposition of the prescription; he defines the context of a prescription as the disjunction of those two propositions, and the avoidance proposition as the negation of the context. Vranas also defines a prescription as being unconditional exactly if its avoidance proposition is (logically) impossible (equivalently, its violation proposition is the negation of its satisfaction proposition), and as being conditional exactly if it is not unconditional. For example, the prescription expressed by “Run” is unconditional and the prescription expressed by “If it rains, run” is conditional; the satisfaction proposition of the latter prescription is the proposition that it rains and you run, the violation proposition is the proposition that it rains and you don’t run, the context is the proposition that it rains, and the avoidance proposition is the proposition that it rains and you don’t run.
doesn't rain. Vranas's account would be rejected by those who deny that prescrip-
tions can ever be avoided.

**Logical Operators and Inconsistency**

This section considers (1) negations, (2) conjunctions and disjunctions, (3) conditionals and biconditionals, and (4) inconsistency.

**Negations**

It is natural to say that the negation of the unconditional prescription expressed by “Dance” is the unconditional prescription expressed by “Don't dance,” and that the negation of the conditional prescription expressed by “If you sing, dance” is the conditional prescription expressed by “If you sing, don't dance.” (One might suggest that the negation of “If you sing, dance” is “Don't both sing and dance,” but this is the negation of “Sing and dance.”) These examples motivate defining the negation of a prescription $I$ as the prescription that is satisfied exactly if $I$ is violated and is violated exactly if $I$ is satisfied – and thus is avoided exactly if $I$ is avoided (Storer 1946: 31). (Assume that specifying satisfaction and violation conditions – and thus also avoidance conditions – uniquely specifies a prescription.) This definition has the consequence that the negation of the negation of a prescription $I$ is just $I$. If, following Vranas (2008), one models $I$ as the ordered pair $<S, V>$ of its satisfaction proposition $S$ and of its violation proposition $V$, then the negation of $I$ is modeled as $<V, S>$.

**Conjunctions and disjunctions**

It is natural to say that the conjunction of the unconditional prescriptions expressed by “Sing” and by “Dance” is the unconditional prescription expressed by “Sing and dance.” This example suggests defining the conjunction of two prescriptions as the prescription that is satisfied exactly if both conjuncts are satisfied and is violated otherwise (see Hofstadter and McKinsey 1939: 448). Although this common definition works when both conjuncts are unconditional, it does not always work: it is natural to say that the conjunction of the conditional prescriptions expressed by “If you sing, dance” and by “If you don’t sing, dance” is the unconditional prescription expressed by “Dance (whether or not you sing),” or just by “Dance,” and this conjunction is satisfied even in some cases in which not both conjuncts are satisfied (for example, if “If you sing, dance” is satisfied – in other words, you sing and you dance – but “If you don’t sing, dance” is avoided – in other words, you sing). To avoid such counterexamples, Vranas (2008: 538–41) proposes the following definition: *the conjunction of two prescriptions is the prescription that is avoided exactly if both conjuncts are avoided and is violated exactly if at least one conjunct is violated* (and, as for every prescription, is satisfied exactly if it is neither violated nor avoided). For example, on this definition the conjunction of “If you sing, dance” and “If you don’t sing, dance” is “Dance,” because “Dance” is avoided exactly if both conjuncts are avoided (that is, exactly if you both don’t sing and sing, so never) and is violated exactly if at least one
imperatives, logic of  5

Having defined conjunction and negation, one can define the disjunction of two prescriptions so that analogues of de Morgan’s laws hold, namely as the negation of the conjunction of the negations of the disjuncts (Vranas 2008: 541–3).

**Conditionals and biconditionals**

The conditional prescription expressed by “If you are happy, smile” can be thought of as a conditional whose antecedent is the proposition expressed by “You are happy” and whose consequent is the prescription expressed by “Smile.” This example motivates defining the *conditional* whose antecedent is a proposition $P$ and whose consequent is a prescription $I$ as the prescription that is satisfied exactly if both $P$ is true and $I$ is satisfied and is violated exactly if both $P$ is true and $I$ is violated (Storer 1946: 31). Following Castañeda (1970: 441–3), one can then define biconditionals by noting that, for example, the prescription expressed by “Smile if and only if you are happy” is the conjunction of the prescriptions expressed by “Smile if you are happy” and by “Smile only if you are happy,” and these two prescriptions are the conditionals that can be expressed respectively by “If you are happy, smile” and by “If you are not happy, don’t smile.”

**Inconsistency**

A common suggestion is that two prescriptions are inconsistent (with each other) exactly if they are *mutually satisfaction-exclusive*; in other words, necessarily, they are not both satisfied. Although this suggestion works when both prescriptions are unconditional (say, “Dance” and “Don’t dance”), it does not always work. The conditional prescriptions expressed by “If you sing, dance” and by “If you don’t sing, don’t dance” are mutually satisfaction-exclusive – necessarily, it is not the case that both (1) you sing and you dance and (2) you don’t sing and you don’t dance – but they are clearly not inconsistent (Castañeda 1970: 443). To avoid this problem, Vranas (2008: 545–6) suggests that two prescriptions are inconsistent exactly if they are *mutually nonviolation-exclusive*; in other words, necessarily, they are not both nonviolated (equivalently: necessarily, at least one of them is violated). (Mutual nonviolation-exclusion entails mutual satisfaction-exclusion and, for *unconditional* prescriptions, it is entailed by it.) On this suggestion, the prescriptions expressed by “If you sing, dance” and by “If you don’t sing, don’t dance” are not inconsistent: if you sing and you dance, then neither prescription is violated. Similarly, however, on this suggestion the prescriptions expressed by “If you sing, dance” and by “If you sing, don’t dance” are not inconsistent, although the one is the *negation* of the other. Vranas (2008: 547) suggests that this result is correct because one can “comply” with both prescriptions by not singing. Still, there is a tension between the two prescriptions, and this can be captured by saying that they are *conditionally inconsistent*: the conjunction of their contexts (namely, the
proposition that you sing) is logically possible and entails that at least one of the two prescriptions is violated.

Imperative Arguments and Imperative Inferences

Distinguish imperative arguments from imperative inferences

An argument can be defined as an ordered pair whose first coordinate is a nonempty set of propositions or prescriptions or both (the premises of the argument) and whose second coordinate is either a proposition or a prescription (the conclusion of the argument). (Alternatively, an argument can be defined in terms of declarative and imperative sentences.) Say that an argument is declarative exactly if its conclusion is a proposition, and that it is imperative exactly if its conclusion is a prescription. Say that an argument is pure exactly if its premises and its conclusion are either all propositions or all prescriptions, and that it is mixed otherwise.

An (endorsing) inference can be defined as a (token) process of reasoning (carried out by a given agent) that starts by endorsing certain propositions or prescriptions, or both (the premises of the inference), and ends by endorsing either a proposition or a prescription (the conclusion of the inference). (Say that to endorse a proposition is to believe that it is true, and that to endorse a prescription is to believe that it is binding. One can similarly define a hypothetical inference by specifying that the premises and the conclusion are not endorsed but are instead entertained only hypothetically.) To any given inference corresponds the argument whose premises and conclusion are the premises and the conclusion of the given inference. Say that an inference is declarative or imperative and pure or mixed exactly if its corresponding argument is. Given these definitions, the rest of this section considers (1) the validity of pure imperative arguments, (2) the validity of mixed arguments, and (3) attacks on the possibility and the usefulness of imperative inferences.

The validity of pure imperative arguments

A reasonable requirement for a definition of validity for pure arguments is that a multiple-premise pure argument $A$ be valid exactly if the corresponding single-premise pure argument is valid whose single premise is the conjunction of the premises of $A$ and whose conclusion is the conclusion of $A$: one should be able to combine multiple premises by conjunction into a single premise and to split a single premise into conjuncts without affecting validity. So in what follows only single-premise pure imperative arguments are considered.

By analogy with the standard definition of validity for pure declarative arguments, one might suggest that a pure imperative argument is valid exactly if some appropriate property is “transmitted” from its premise to its conclusion. The appropriate property cannot be truth (which is “transmitted” in valid pure declarative arguments), but it can be (i) satisfaction, (ii) nonviolation, or (iii) bindingness. So, say that a pure imperative argument is satisfaction-valid exactly if, necessarily, its conclusion is satisfied if its premise is (Hofstadter and McKinsey 1939: 452); and
define *nonviolation-valid* and *bindingness-valid* similarly. (i) Against satisfaction validity, it can be shown that the argument from “If your Graduate Record Examination (GRE) score is at least 600, apply” to “If your GRE score is at least 700, apply” (which is intuitively valid: think about instructions on a graduate admissions website) is not satisfaction-valid, and that the argument from “If your GRE score is at least 600, apply” to “If your GRE score is at least 500, apply” (which is intuitively invalid) is satisfaction-valid. (ii) Against nonviolation validity, it can be shown that the arguments from “Apply” to “If you don’t apply, kill yourself” and from “If you have a perfect score, apply” to “If you don’t apply, don’t have a perfect score” (which are arguably intuitively invalid) are nonviolation-valid. (iii) Against bindingness validity, note that, in the absence of a formal account of bindingness, it is unclear how to find out whether any given pure imperative argument is bindingness-valid.

Vranas (2011) defines when a reason “weakly” or “strongly” supports a prescription (see REASONS), and he defines a pure imperative argument as being weakly valid exactly if, necessarily, every reason that weakly supports the premise also weakly supports the conclusion (he defines strongly valid similarly). Vranas argues that there is an unresolvable conflict of intuitions about the validity of the argument (which corresponds to “Ross’s paradox”; see Ross 1941: 62) from “Post the letter” to “Post the letter or burn it,” and that this conflict is largely explained by a divergence between strong and weak validity: the argument is weakly but not strongly valid. Against weak validity, it can be shown that the argument from “Apply” to “If you don’t apply, kill yourself” (which is arguably intuitively invalid) is weakly valid.

For further definitions of validity for pure imperative arguments, see Hare (1952: 25; see HARE, R. M.), Rescher (1966: 82–91), Sosa (1966: 232), and Makinson and van der Torre (2000).

**The validity of mixed arguments**

Concerning mixed *declarative* arguments (namely arguments whose conclusion is a proposition and whose premises include a prescription), one might propose the principle (which corresponds to “Hare’s Thesis”; see Rescher 1966: 73) that such an argument is valid only if its conclusion follows from its declarative premises alone. Against this principle, one might claim that the arguments from “Kiss Paul’s sister” to “Paul has a sister” (see Rescher 1966: 92–7) and from “If smoking is permitted, smoke” and “Don’t smoke” to “Smoking is not permitted” (see Geach 1958: 52) are valid. In reply, one might claim that “Kiss Paul’s sister” presupposes (but does not entail) “Paul has a sister,” and that the conjunction of “If smoking is permitted, smoke” and “Don’t smoke” – which on Vranas’s definition of conjunction (see above) is “Let it be the case that smoking is not permitted and that you don’t smoke” – entails “Let it be the case that smoking is not permitted,” not “Smoking is not permitted.”

Concerning mixed *imperative* arguments with only declarative premises, namely arguments from only propositions to a prescription, one might propose the principle (which corresponds to “Poincaré’s Principle”; see Rescher 1966: 74) that no
such argument is valid (except perhaps trivially, if its conclusion is vacuous – namely necessarily satisfied, like “Run or don't run” – or if its premises are inconsistent). This principle is similar to – but different from – the is/ought thesis attributed to Hume (see IS–OUGHT GAP; HUME, DAVID). Against this principle, Vranas (2015) claims that the argument from “There is an undefeated reason for you to tell the truth” to “Tell the truth” is valid. More generally, Vranas proposes that a proposition entails a prescription exactly if the proposition entails that some reason undefeatedly supports the prescription.

Finally, concerning mixed imperative arguments with both declarative and imperative premises, one might propose that such an argument is valid exactly if, necessarily, if its declarative premises are true and its imperative premises are satisfied, then its (imperative) conclusion is satisfied. This proposal has the implausible consequence that the argument from “Smile” and “You will run” to “Run” is valid. Vranas (2015) proposes instead that such an argument is valid exactly if, necessarily, every reason that both “guarantees” (in a sense defined by Vranas) its declarative premises and supports its imperative premises also supports its (imperative) conclusion. This proposal has the controversial consequence that the argument from “If you drink, don't drive” and “You are going to drink” to “Don't drive” is invalid. For further proposals concerning the validity of such arguments, see Clarke (1970), Sosa (1970), and Parsons (2013).

**Attacks on imperative inference**

Although imperative logic deals with imperative arguments rather than with imperative inferences, if imperative inferences were impossible, then what would be the point of imperative logic? Contesting the possibility of imperative inferences, Bernard Williams (1963; see WILLIAMS, BERNARD) gives an argument that can be reconstructed as follows: distinct prescriptions have conflicting permissive presuppositions (e.g., “Sing” does not permit dancing without singing, but “Sing or dance” does permit this); thus successively endorsing the premises and the conclusion of a pure imperative argument (e.g., the argument from “Sing” to “Sing or dance”) amounts to changing one's mind and cannot amount to carrying out an inference. Williams's reasoning is widely discussed in the literature (for references, see Vranas 2010: 66) and is subject to several objections. For example, Vranas (2010: 67) argues that one can carry out an inference even if, while doing so, one changes one's mind, as illustrated by an examiner who says “Answer exactly two out of the three questions – any two questions, at your choice” and a minute later says “I changed my mind. Don't answer both the first and the last question. Therefore, answer either the first two or the last two questions (not both), at your choice.”

Even if one grants that imperative inferences are possible, one might argue that they are useless because they can always be replaced with declarative inferences, in the sense that the validity of an imperative argument always amounts to the validity of a corresponding declarative argument (see Harrison 1991: 116); for example, the validity of the imperative argument from “Sing” to “Sing or dance” amounts to
the validity of the declarative argument from “You will sing” to “You will sing or dance.” Vranas (2010: 69) replies with an analogy: even if geometric reasoning can always be replaced with algebraic reasoning (by using Cartesian coordinates), geometric reasoning is often useful, because it is often more straightforward than corresponding algebraic reasoning. Similarly, Vranas argues that imperative reasoning is often more straightforward than corresponding declarative reasoning.

For further attacks on imperative inference, see Wedeking (1970), Harrison (1991), and Hansen (2008); for replies, see Castañeda (1971) and Vranas (2010).

See also: Deontic logic; Frege–Geach objection; Hare, R. M.; Hume, David; is–ought gap; metaethics; non-cognitivism; prescriptivism; reasons; Williams, Bernard

REFERENCES

10 IMPERATIVES, LOGIC OF


FURTHER READINGS


